Computing Replenishment Cycle Policy Parameters for a Perishable Item

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Food Supply Chain Management

"The **availability** and the **assortment** of fresh products have a significant influence on **consumer choice**"

Dawson (2004)

Food supply chain management has an increasingly strategic importance in retail competition.

Inventory Control of Perishables

"In available Advanced Planning Systems shelf-life aspects of food are **not adequately incorporated**"

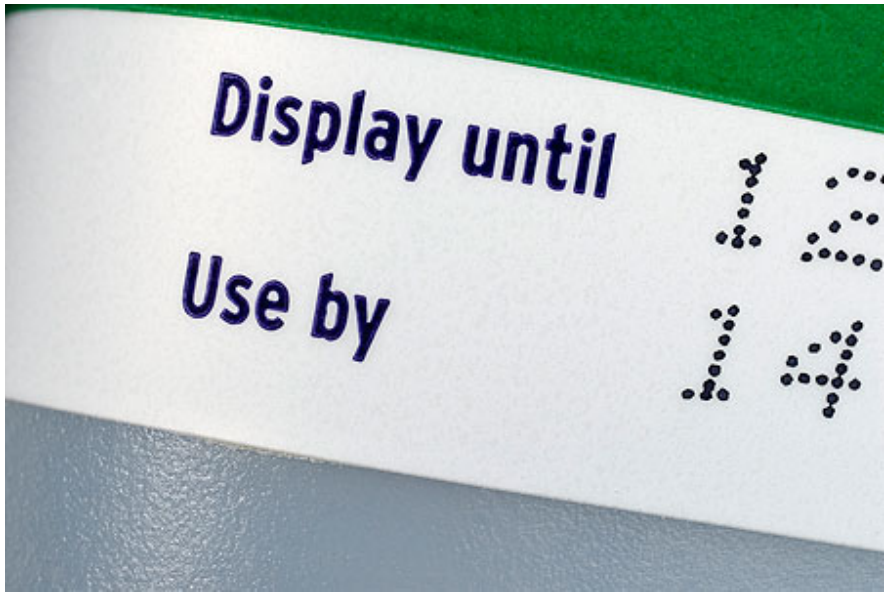
Lütke Entrup (2005)

Inventory Control of Perishables

This is mainly due to the fact that inventory control of perishable products is, in general, a **challenging task.**

Open Issues

Every month we observe billions of dollars of food **expiring** on supermarket shelves…



Open Issues

…at the same time supermarkets **lose revenues** for food products that are **not available** on their shelves.

Some Data

Gruen et al. (2002) report that the average out-of-stock rate is

Worldwide:	8.3%
Europe:	8.6%
US:	7.9%

Costs Trade-off

This situation reflects the classic **newsvendor trade-off** between

- ordering too much and observing **wastage**, or
- ordering too little and facing **lost sales**.

Existing Works

Inventory problems of **perishable products** have been discussed extensively in the literature.

The first and most cited review work on this topic is by **Nahmias (1982)**.

Perishable Inventory Theory: A Review

STEVEN NAHMIAS
The University of Santa Clara, Santa Clara, California
(Received February 1980; accepted January 1982)

This paper reviews the relevant literature on the problem of determining suitable ordering policies for both fixed life perishable inventory, and inventory subject to continuous exponential decay. We consider both deterministic and stochastic demand for single and multiple products. Both optimal and sub-optimal order policies are discussed. In addition, a brief review of the application of these models to blood bank management is included. The review concludes with a discussion of some of the interesting open research questions in the area.

THIS ARTICLE reviews the current literature on ordering policies for perishable inventories. Most inventory models assume that stock items can be stored indefinitely to meet future demands. However, certain types of inventories undergo change in storage so that in time they may become partially or entirely unfit for consumption. For example, lysis of red blood cells renders blood unacceptable for transfusion 21 days after it is drawn. Fresh produce, meats and other foodstuffs become unusable after a certain time has elapsed. Photographic film and drugs are further examples of items which have a limited useful lifetime.

We will consider two classifications of perishability: fixed lifetime and random lifetime. The former category includes those cases where the lifetime is known *a priori* to be a specified number of periods or a length of time independent of all other parameters of the system. As we will show, the latter category will include exponential decay as a special case and will also include those cases where the product lifetime is a random variable with a specified probability distribution. (Van Zyl [1964] has used the classification "age dependent" perishability and "age independent" perishability to distinguish between fixed life and exponential decay. Our scheme is broader; it includes other random lifetime models.)

The review is organized as follows:

I. Fixed Life Perishability
  1. Deterministic Demand
  2. Stochastic Demand
    a. Optimal Policies for a Single Product
    b. Approximately Optimal Policies for a Single Product

Subject classification: 331 inventory production, 347 inventory/production: perishable items.

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Recent Works

A more recent work is by **Karaesmen et al. (2008)**, who review the supply chain management literature of perishable products having **fixed** or **random lifetimes**.

Chapter XX

MANAGING PERISHABLE AND AGING INVENTORIES: REVIEW AND FUTURE RESEARCH DIRECTIONS

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ABSTRACT

We review the literature on supply chain management of perishable products that have a fixed or random lifetime, primarily focusing on challenges introduced by the *aging* of such products. We start with a review of the literature on capacity and production planning, and then move to the literature on inventory management at a single location. The research on single-location models provides a basis for research on multi-echelon/multi-location models and transshipment/distribution models, which are also reviewed in this chapter. We emphasize research involving modeling novelties, in terms of demand-fulfillment, information sharing, and centralized vs. decentralized planning. We provide several directions for future research by identifying the gaps in the literature as well as the needs of industries where perishable products are commonly stocked.

Key Issues

Typically a **total profit maximization or total cost minimization** approach is pursued.

However, the **structure of the optimal replenishment policy** is typically **complex**:

- the replenishment quantity depends on the **individual age categories** of current inventories and **all outstanding orders**.

Key Issues

Optimal ordering policies are **not base-stock policies** or, in general, **easy to implement**.

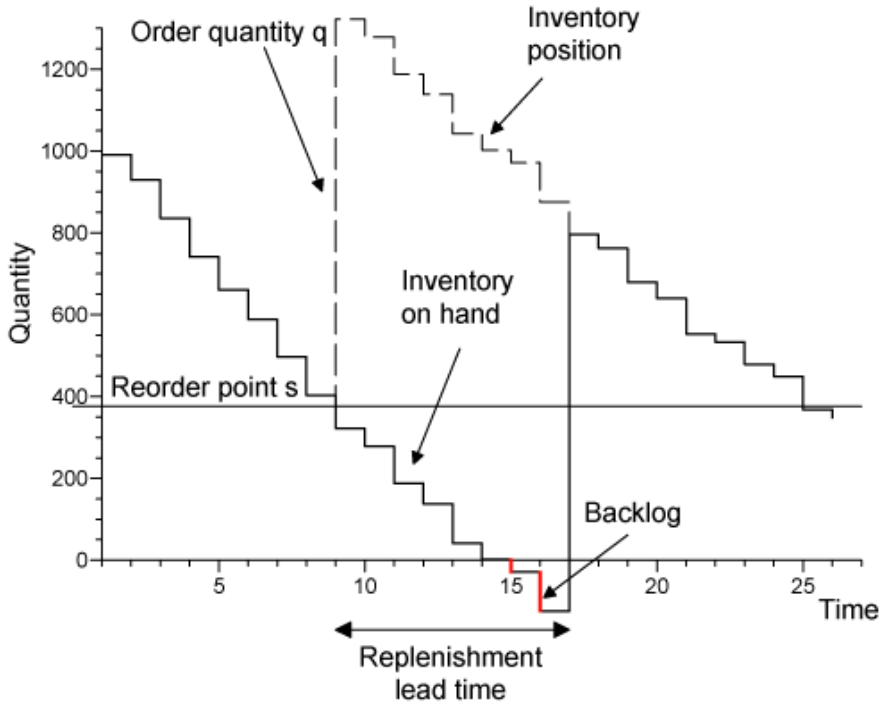
see e.g. van Zyl (1964)

Therefore, simplifying **heuristic ordering policies** have received much attention

Cohen (1976)

Brodheim et al. (1975)

Pierskalla and Roach (1975)



Key Issues

According to Karaesmen et al. (2008) all the existing works **ignore fixed ordering costs**.

Relaxing such an assumption clearly may only **complicate further** the **structure** of the optimal policy.

For this reason developing **effective heuristic** policies is **of great practical importance** in inventory systems for perishable items in which a **fixed ordering cost** is considered.

Key Issues

Nahmias (1978) points out that in practice it is far **more realistic** to allow the ordering cost to be composed of both f**ixed** and **proportional** components.

He considers a problem in which the so-called "standard" cost structure is adopted, which is composed by **fixed** and **variable** ordering costs, **wastage** costs, **holding** and **shortage** costs.

Key Issues

However, nowadays it is widely known and accepted that **stock-out penalty cost** approaches are

- **difficult to implement** and
- **hardly used in practice**,

mainly due to the difficulties in the estimation of their value, e.g., how to quantify the loss of goodwill?

Key Issues

Therefore, **a service level approach** becomes essential for fresh food inventory management.

However, according to the two surveys mentioned above, in the inventory control literature, this approach has been **rarely analyzed** so far.

Key Issues

One of the pioneering works in the application of **a service level oriented approach** in inventory control of perishable products is by Minner and Transchel (2010)

They discuss a periodic-review, single-product service-level approach for perishable item inventory control with positive lead times, fixed shelf-life, different inventory issuing policies, and non-stationary demands under the lost sales assumption.

Nevertheless, one of the simplifying assumptions adopted in this work is that **the fixed ordering costs are negligible**.

Our Research

In this work, we analyze a **periodic-review**, **single-location**, **single-product** production/ inventory control problem under **non-stationary stochastic demand** and **service level constraints** for a **perishable item**.

In contrast to Minner and Transchel (2010), who assume lost sales, we operate under a **complete back-order** assumption. Our cost structure considers **fixed** and **variable ordering costs**, **wastage costs** and **holding costs**.

A Stochastic Programming Model (sketch)

$$\min \int_{d_1} \dots \int_{d_N} \sum_{t=1}^{N} \left(a\delta_t + h \sum_{i=0}^{M-1} I_t^i + uQ + wI_t^M \right) g(d_1) \dots g(d_N) \mathrm{d}d_1 \dots \mathrm{d}d_N$$

subject to

$$\delta_t = \begin{cases} 1 & Q_t > 0 \\ 0 & otherwise \end{cases} \qquad t = 1, \dots, N$$

$$\sum_{i=0}^{M} I_t^i + d_t - \sum_{i=0}^{M-1} I_{t-1}^i = Q_t \qquad t = 1, \dots, N$$

$$I_t^i = \max\left\{ I_{t-1}^{i-1} - \max\left\{ d_t - \sum_{j=i}^{M-2} I_{t-1}^j, 0 \right\}, 0 \right\} \qquad t = 1, \dots, N \quad i = 1, \dots, M$$

$$\Pr\left\{ \sum_{i=0}^{M} I_t^i \geq 0 \right\} \geq \alpha \qquad t = 1, \dots, N$$

$$I_0^i = 0 \qquad i = 0, \dots, M-1$$

$$I_t^i \geq 0 \qquad t = 1, \dots, N \quad i = 1, \dots, M$$

$$I_t^0 \in R \qquad t = 1, \dots, N$$

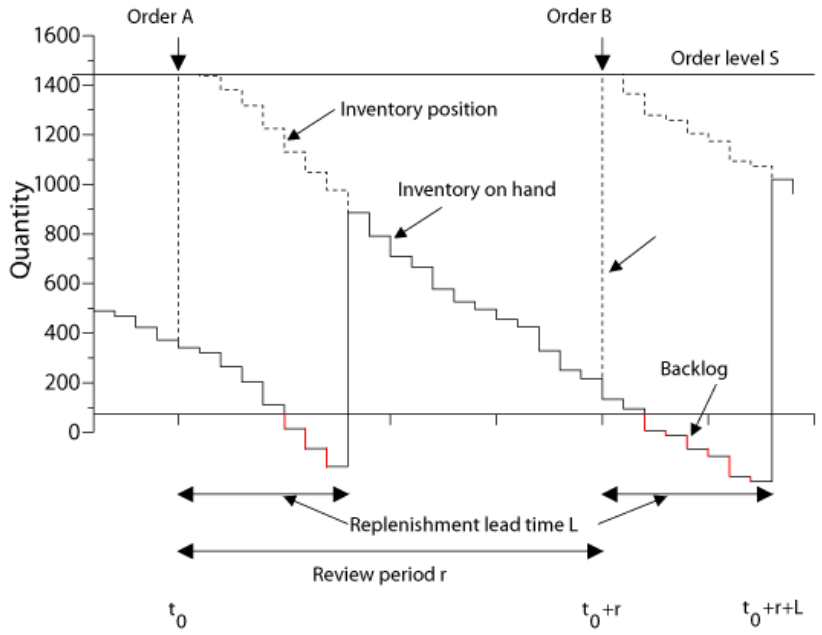
$$\delta_t \in \{0,1\} \qquad t = 1, \dots, N$$

$$Q_t \geq 0 \qquad t = 1, \dots, N$$

Replenishment Cycle Policy

We propose an **easy to implement** non-stationary "replenishment cycle" policy, i.e. (Rn,Sn)

This policy yields at most *2N* **control parameters**.



Why (R,S)?

"It is pointed out that in many logistics systems inventory replenishment by retailers consists of **two distinct stages**:

1. in the first stage, retailers may make a **delivery request** in advance without making a firm commitment on order quantity.

2. in the second stage, the retailers are allowed to make an **adjustment to the quantity requested** right before the shipping schedule is finalized.

This approach allows retailers to **postpone their quantity decision** until **better demand information** is available, and, as a result, the mismatch between supply and demand may be reduced."

Li et al. (2009)

Deterministic-equivalent CP model

$$\min \sum_{t=1}^{N}\left(a\delta_t + h\sum_{i=0}^{M-1}\tilde{I}_t^i + w\tilde{I}_t^M \right) + u\sum_{i=0}^{M}\tilde{I}_N^i$$

subject to

$$\left(\sum_{i=0}^{M}\tilde{I}_t^i + \tilde{d}_t - \sum_{i=0}^{M-1}\tilde{I}_{t-1}^i\right) \Rightarrow \delta_t = 0 \qquad t = 1,\ldots,N$$

$$\sum_{i=0}^{M}\tilde{I}_t^i + \tilde{d}_t - \sum_{i=0}^{M-1}\tilde{I}_{t-1}^i \geq 0 \qquad t = 1,\ldots,N$$

$$\tilde{I}_t^i = \max\left\{\tilde{I}_{t-1}^{i-1} - \max\left\{\tilde{d}_t - \sum_{j=i}^{M-2}\tilde{I}_{t-1}^j, 0\right\}, 0\right\} \qquad t = 1,\ldots,N \quad i = 1,\ldots,M$$

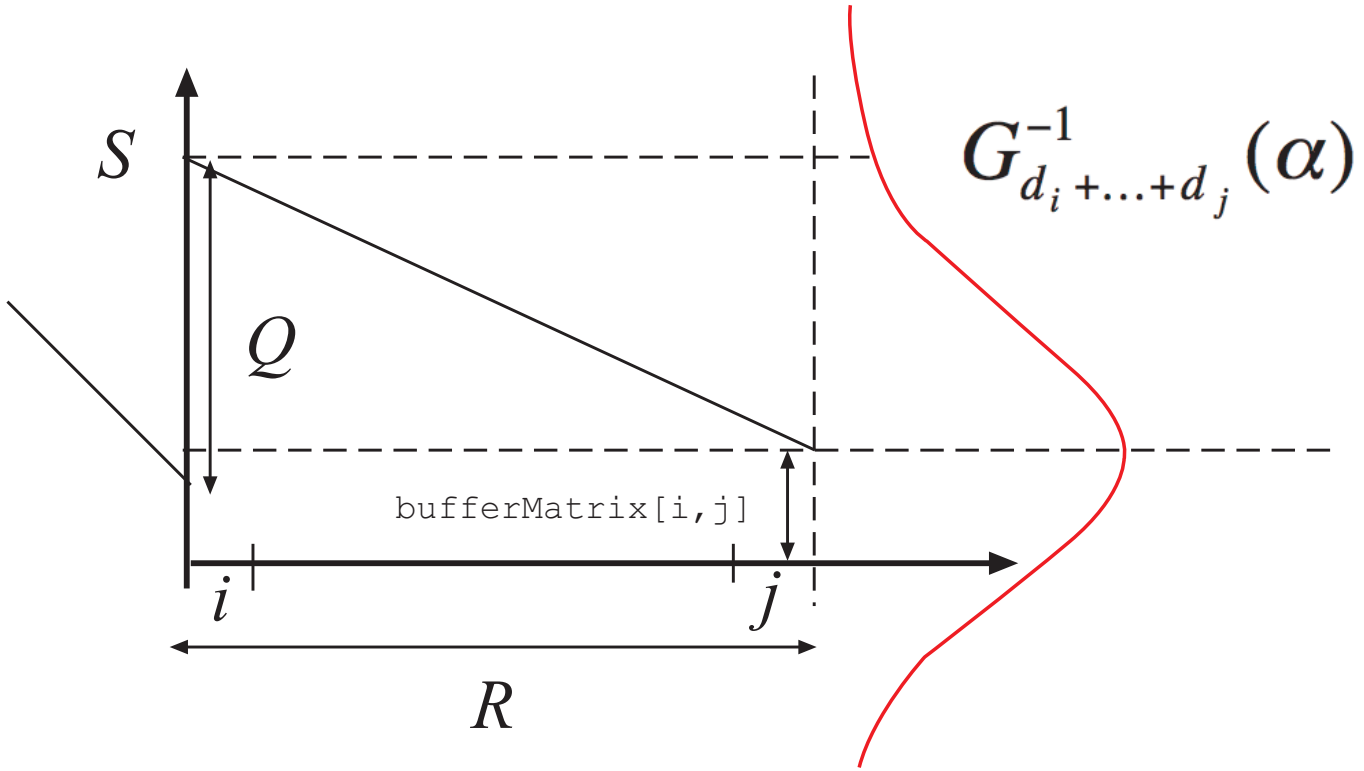
$$\sum_{j=i}^{M}\tilde{I}_t^j \geq \text{bufferMatrix}\left[\max_{j\in\{1,\ldots,t\}} j\delta_j, t\right] \qquad t = 1,\ldots,N$$

$$\tilde{I}_0^i = 0 \qquad i = 0,\ldots,M$$

$$\tilde{I}_t^i \geq 0 \qquad t = 1,\ldots,N \quad i = 0,\ldots,M$$

$$\delta_t \in \{0,1\} \qquad t = 1,\ldots,N$$

Service Level Constraints



where

$$\text{bufferMatrix}[i,j] = G^{-1}_{d_i+\ldots+d_j}(\alpha) - \sum_{k=i}^{j} \tilde{d}_k$$

and $G_{d_i+\ldots+d_j}(.)$ is the cumulative distribution function of $d_i+\ldots+d_j$.

Service Level Constraints

- Also quite readable
- Efficient implementation via the `element(X,Y[],Z)` global constraint

$$\sum_{j=i}^{M} \tilde{I}_t^j \geq \text{bufferMatrix}\left[\max_{j\in\{1,\ldots,t\}} j\delta_j, t\right] \qquad t = 1,\ldots,N$$

Age Transition Constraints

- FIFO issuing rule
- Easy to grasp
- Higher expressiveness compared to MIP

$$\tilde{I}_t^i = \max\left\{\tilde{I}_{t-1}^{i-1} - \max\left\{\tilde{d}_t - \sum_{j=i}^{M-2}\tilde{I}_{t-1}^j, 0\right\}, 0\right\} \quad t = 1,\ldots,N \qquad i = 1,\ldots,M$$

- A LIFO rule is also very easy to express…

Numerical Example

An **8-period** planning horizon is considered

$\tilde{d}_t = \{100,125,25,40,30,80,110,50\}$

Lifetime of 4 periods: $M = 3$

We assume an initial null inventory level and a **normally distributed demand** for every period with a coefficient of variation $\sigma_t / \tilde{d}_t = 0.2$

The other problem **parameters** are:

$a = 100 \quad u = 5 \quad w = 2 \quad h = 1 \quad \alpha = 0.95$

Optimal (R,S) Policy

Cost: 962

Table 1. Policy parameters

Period	1	2	3	4	5	6	7	8
$\tilde{d}_t$	100	125	25	40	30	80	110	50
S_t	133	192	-	86	-	106	146	66
$\tilde{I}_t^0$	33	67	0	44	2	26	36	16
$\tilde{I}_t^1$	0	0	42	0	14	0	0	0
$\tilde{I}_t^2$	0	0	0	2	0	0	0	0
$\tilde{I}_t^3$	0	0	0	0	0	0	0	0

Implementing non-stationary (R,S) for this instance brings a **50% cost benefit** w.r.t. stationary (R,S) – widely used in practice.

Conclusions

We discussed a **Constraint Programming** deterministic equivalent model for computing non-stationary (R,S) policy parameters for a perishable item under a "standard" cost structure and service level constraints

Our approach is particularly suitable for being implemented in practice due to the **simplicity of the policy** proposed (*2N* policy parameters)

Our model represents one of the first attempts to include both **a fixed ordering cost** and **service level constraints** in an inventory model for perishable items

Future Research

- Comparison with the **optimal policy** for the problem
  - Policy computed by DP?

    see Haijema et al. 2009

- Comparison with **other policies** that are typically adopted by practitioners

- Efficient **computation methods** to enhance our basic model

- Proving that **the structure** we adopted for the non-stationary (R,S) policy (with 2N policy parameters) is **optimal for a perishable inventory**.

Questions