Routing decisions of a hybrid vehicle on electric road networks

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Decarbonising transportation

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Global Greenhouse Gas Emissions by the Transportation Sector

Decarbonising transportation

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A solution: Road electrification
Overhead catenary systems and compatible vehicles

- Seamless connection
- The system powers the vehicle
- The system charges its battery
- Range extender on the vehicle (e.g., diesel engine)
Electrification plan in the UK

**PHASE 1**
- Distance: 2,026 miles
- Construction time: 2 years
- Cost: £5.6 billion
- HGV km coverage: 31%

**PHASE 2**
- Distance: 2,638 miles
- Construction time: 2.6 years
- Cost: £5.1 billion
- HGV km coverage: 50%

**PHASE 3**
- Distance: 3,914 miles
- Construction time: 2.5 years
- Cost: £7.1 billion
- HGV km coverage: 65%
Problem description I

Some assumptions:

- Single vehicle
- Loading inventory at depot location (single product)
- Visiting retailers across the network
- The vehicle is a hybrid HGV (using fuel only if battery is depleted)
- Minimising: (1) electricity and fuel energy costs (2) (expected) lost sales costs
Shortest paths for EVs on electrified roads

- Battery capacity of vehicle is 30 units
- Energy cost per unit from electricity/battery: 1
- Energy cost per unit from fuel: 2
- The vehicle uses battery whenever is possible
- How to go from node 0 to node 2?
Shortest paths for EVs on electrified roads

- Battery capacity of vehicle is 30 units
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Problem description II

- Graph $G = \langle \mathcal{N}, \mathcal{A} \rangle$ representing the road network
- Depot: node 0, Retailers: $\mathcal{C} \subseteq \mathcal{N}$
- Discrete time horizon $T$ periods
  - Stochastic demand $d^c_t$
  - Vehicle location: $V^i_t$ (binary var.)
  - Vehicle load-up & delivery to $c$: $L_t$ & $Q^i_t$
- Customers capacity: $k_c$, vehicle capacity: $K$
- Required & supplied battery $(i, j)$: $r_{ij}(M)$ & $s_{ij}$
- $C^b, C^f$: kWh cost of electric road, battery, or fuel
- Lost sales penalty per unit: $p$
Energy model

Power:

\[ P(a, v) = Mav + Mg \sin \theta + 0.5 C_d A \rho v^3 + MgC_r \cos \theta v \] (1)

Required energy arc \((i, j)\)

\[ r_{ij}(M) = \lambda P(0, v_{ij})(d_{ij}/v_{ij}) = \lambda (Mg \sin \theta_{ij} + 0.5 C_d A \rho v_{ij}^2 + MgC_r \cos \theta_{ij})d_{ij} = \alpha_{ij} M + \beta_{ij} \] (2)

where \(\alpha_{ij} = \lambda d_{ij} g(\sin \theta_{ij} + C_r \cos \theta_{ij})\) and \(\beta_{ij} = \lambda d_{ij} 0.5 C_d A \rho v_{ij}^2\) are arc constants.
A simple example

- Depot: node 0, Customer nodes: 1, 2, $k_1 = k_2 = 5$
- Demand $d_{t}^{c} = 1 \forall c, t$; Lost sales $p = 25$
- Vehicle weight $w = 1$, battery cap. $B = 20$, inv. cap. $K = 4$
- $C^b = 1$ and $C^f = 5$
- Start: $V_0^0 = 1$, $L_0 = 0$ $b_0 = 0$

Tuples on arcs are $(\alpha_{ij}, \beta_{ij})$
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![Graph showing a network with nodes and tuples on arcs]
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- \( C^b = 1 \) and \( C^f = 5 \)
- Start: \( V_0^0 = 1, L_0 = 0, b_0 = 0 \)

Figure 1: Problem instance on a ERS network. Tuples on arcs are \( (\alpha_{ij}, \beta_{ij}); \) Supplied energy in the blue arc is \( s_{04} = 20, \) while the rest are zero.
Stochastic MILP approximation

▶ Static uncertainty strategy
▶ (R,Q) policy: replenishments and delivery quantities are fixed at the beginning of the planning horizon
▶ The solution gives a fixed route
▶ Energy costs are deterministic for the model

\[
\min \sum_{t=2}^{T} \sum_{i=1}^{N} \sum_{j=1}^{N} C^b s_{ij} T_{t-1}^{ij} + \sum_{t=2}^{T} C^b E^b_t + C^f E^f_t + \sum_{t=1}^{T} \sum_{i=1}^{C} p[l^i_t]^-
\] (3)
Inventory constraints

Definition

Given a random variable $\omega$ and a scalar $q$, the first order loss function is defined as:

$$\mathcal{L}_\omega(q) = E[\max(\omega - q, 0)]$$

Reciprocally, the complementary first order loss function is:

$$\hat{\mathcal{L}}_\omega(q) = E[\max(q - \omega, 0)]$$

Constraints:

$$[l_{it}^-] = \mathcal{L}_{d_{it}} \left( s_i + \sum_{k=1}^{t} Q_i^k + \sum_{k=1}^{t-1} [l_{t-1}^-] - \sum_{k=1}^{t} [E_i^k] \right) t = 1, \ldots, T; i = 1, \ldots, C$$

$$[l_{it}^+] = \hat{\mathcal{L}}_{d_{it}} \left( s_i + \sum_{k=1}^{t} Q_i^k + \sum_{k=1}^{t-1} [l_{t-1}^-] - \sum_{k=1}^{t} [E_i^k] \right) t = 1, \ldots, T; i = 1, \ldots, C$$

$$[E_i^t] = \max \left( [l_{it}^+] + Q_i^t - s_i, 0 \right) t = 1, \ldots, T; i = 1, \ldots, C$$

Linearisation of the loss function:

Numerical experiments: testbed design

Initial inventory at \{R_1, R_2\}

- Demand distributions
  - (D1) \(\lambda_{R1} = \{2,2,2,2,2,2,2,2,2\}\), \(\lambda_{R2} = \{2,2,2,2,2,2,2,2,2\}\)
  - (D2) \(\lambda_{R1} = \{1,1,2,2,3,3,4,4,5\}\), \(\lambda_{R2} = \{5,4,4,3,3,2,2,1,1\}\)
  - (D3) \(\lambda_{R1} = \{1, 1, 1, 1, 1, 2, 2, 2, 3, 1\}\), \(\lambda_{R2} = \{1, 1, 2, 1, 1, 2, 2, 3, 1\}\)

- Unit penalty cost \(p = \{10, 20, 30\}\)
**Numerical experiments: testbed design**

**Table:** Pivot table of mean, median and standard deviation of percentage error (MPE, MdPE, SD respectively) of the solutions obtained by the MILP heuristic for the computational study

<table>
<thead>
<tr>
<th></th>
<th>MPE</th>
<th>MdPE</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Network</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>3.47%</td>
<td>3.42%</td>
<td>2.14%</td>
</tr>
<tr>
<td>T2</td>
<td>3.48%</td>
<td>3.19%</td>
<td>2.05%</td>
</tr>
<tr>
<td>T3</td>
<td>3.18%</td>
<td>2.53%</td>
<td>2.57%</td>
</tr>
<tr>
<td>T4</td>
<td>4.73%</td>
<td>3.88%</td>
<td>4.12%</td>
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<tr>
<td><strong>Initial inv.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,0)</td>
<td>3.94%</td>
<td>3.41%</td>
<td>2.87%</td>
</tr>
<tr>
<td>(5,5)</td>
<td>3.49%</td>
<td>3.03%</td>
<td>2.91%</td>
</tr>
<tr>
<td><strong>Penalty</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.67%</td>
<td>1.30%</td>
<td>1.57%</td>
</tr>
<tr>
<td>20</td>
<td>3.98%</td>
<td>3.51%</td>
<td>2.45%</td>
</tr>
<tr>
<td>30</td>
<td>5.50%</td>
<td>5.22%</td>
<td>3.06%</td>
</tr>
<tr>
<td><strong>Demand pattern</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D1</td>
<td>3.70%</td>
<td>3.39%</td>
<td>2.67%</td>
</tr>
<tr>
<td>D2</td>
<td>3.50%</td>
<td>3.02%</td>
<td>2.71%</td>
</tr>
<tr>
<td>D3</td>
<td>3.95%</td>
<td>3.29%</td>
<td>3.28%</td>
</tr>
<tr>
<td><strong>General</strong></td>
<td>3.72%</td>
<td>3.25%</td>
<td>2.90%</td>
</tr>
</tbody>
</table>
Electrification stages example

<table>
<thead>
<tr>
<th>Instance</th>
<th>Stage 0 Battery cost</th>
<th>Stage 0 Fuel cost</th>
<th>Stage 1 Battery cost</th>
<th>Stage 1 Fuel cost</th>
<th>Stage 2 Battery cost</th>
<th>Stage 2 Fuel cost</th>
<th>Stage 3 Battery cost</th>
<th>Stage 3 Fuel cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>50.00</td>
<td>87.38</td>
<td>50.00</td>
<td>87.38</td>
<td>121.40</td>
<td>9.53</td>
<td>127.34</td>
<td>9.53</td>
</tr>
<tr>
<td>R2</td>
<td>50.00</td>
<td>131.01</td>
<td>122.50</td>
<td>17.77</td>
<td>118.17</td>
<td>0.00</td>
<td>125.42</td>
<td>0.00</td>
</tr>
<tr>
<td>R3</td>
<td>50.00</td>
<td>148.34</td>
<td>91.06</td>
<td>99.89</td>
<td>165.66</td>
<td>11.29</td>
<td>176.68</td>
<td>0.00</td>
</tr>
<tr>
<td>R4</td>
<td>50.00</td>
<td>84.15</td>
<td>79.11</td>
<td>32.70</td>
<td>102.73</td>
<td>0.00</td>
<td>108.66</td>
<td>0.00</td>
</tr>
<tr>
<td>R5</td>
<td>50.00</td>
<td>126.90</td>
<td>110.71</td>
<td>22.33</td>
<td>110.71</td>
<td>22.33</td>
<td>110.71</td>
<td>22.33</td>
</tr>
<tr>
<td>R6</td>
<td>50.00</td>
<td>67.86</td>
<td>112.34</td>
<td>0.00</td>
<td>106.69</td>
<td>0.00</td>
<td>119.67</td>
<td>0.00</td>
</tr>
<tr>
<td>R7</td>
<td>50.00</td>
<td>101.27</td>
<td>88.07</td>
<td>20.30</td>
<td>88.07</td>
<td>20.30</td>
<td>102.09</td>
<td>0.00</td>
</tr>
<tr>
<td>R8</td>
<td>50.00</td>
<td>74.02</td>
<td>86.63</td>
<td>0.00</td>
<td>86.63</td>
<td>0.00</td>
<td>86.63</td>
<td>0.00</td>
</tr>
<tr>
<td>R9</td>
<td>50.00</td>
<td>166.85</td>
<td>120.80</td>
<td>36.55</td>
<td>121.36</td>
<td>17.91</td>
<td>134.38</td>
<td>0.00</td>
</tr>
<tr>
<td>R10</td>
<td>50.00</td>
<td>176.37</td>
<td>176.37</td>
<td>0.00</td>
<td>176.37</td>
<td>0.00</td>
<td>176.37</td>
<td>0.00</td>
</tr>
</tbody>
</table>

% costs from fuel:
- Stage 0: 69.95%
- Stage 1: 35.12%
- Stage 2: 11.75%
- Stage 3: 2.52%