Introduction

Figure 1: The Norns are female beings who weave the loom of life and spin the web of wyrd — and thus the destiny of gods and men — into existence. Similar triads, such as the Latin Fates or the Greek Moirai, are found all across European mythology. In a word, the Norns were the solution ancient people adopted to represent change.

We shall begin our discussion with a question:

> Decision-making under uncertainty: Is there any other kind?
>
> Naomi Oreskes (University of California, San Diego, US)

Following Chacko, who defines decision-making as: "the commitment of resources today for results tomorrow" [Chacko, 1991, p. 5], Oreskes posits that because decisions involve expectations about the future, they *always* involve uncertainty.

Further hints about the *irreducibility of uncertainty* can be obtained by analysing its antonym: *certainty*.

> Certain is an ambiguous word; it can mean sure, but it can also mean particular. Uncertain does not have this ambiguity; unsure being a close synonym.[2]

[2] D. V. Lindley. *Understanding uncertainty*. Wiley-Interscience, Hoboken, N.J, 2006

"Certain" comes from certus, the past participle of cernere: "to distinguish, decide;" literally "to sift, separate." The associated Proto-Indo-European root is *krei-: "to sieve," thus "discriminate, distinguish," which is also the source of Ancient Greek κρίσις: "turning point, judgment, result of a trial." From which the English word "crisis" also originates, albeit with a slightly different meaning that relates to the trouble one experiences in the decision making process.

''Sure" comes from Latin securus. Literally, carefree: from sē- ("apart") + cūra ("care"). Essentially, free from having to take a decision.

Uncertain is therefore the state of an agent who is at a *turning point* and is about to make a decision — *decision making and uncertainty are deeply intertwined.*

It is worth observing that Ancient Greeks had a name for such a turning point: καιρός. Kairos denotes the right, critical, or opportune moment: the moment at which a decision had to be taken.[3] Conversely, χρόνος refers to chronological or sequential time.[4] Καιρός was typically represented as a young and beautiful god standing on tiptoe on a rolling ball, because he is always running. He represents the "fleeting moment," which must be grasped by the tuft of hair on the personified forehead of the fleeting opportunity; otherwise the moment is gone and can not be re-captured — personified by the back of the boy head being bald, and thus offering no further opportunity to grasp. Immediately behind Kairos, these representations typically feature a desperate figure who follows: μετανοια, the representation of *afterthought* — or, in modern terms, of buyer's remorse (Fig. 2).

> Metanoia exacts punishment for what has and has not been done, so that people regrets it.
>
> Ausonius

No matter what one chooses, an alternative always exists, otherwise there would be no choice. It is this (or these) alternative(s), which generates regret. Thus, *regret* is an intrinsic part of decision making.

Unfortunately, one may say that not all uncertainties are created equal: not only there are multiple notions of uncertainty in use, but these notions have dramatically changed over time.[5] Cline provides a historical account on the evolution of the concept of risk by focusing on its etymology. In particular, he discusses the transition that occurred at the beginning of the 16th century, when Europeans started to understand that, by relying on technology, they could — at least partially — influence their own future. For instance, cartography and compass helped navigating the sea, whilst double-entry bookkeeping helped keeping track of profit and losses over the years.[6] This marked the beginning of the end for the old Europe-wide concept of pre-ordained fate, controlled by the Moirai, Fates, or Norns (Figure 1); and paved the way towards the revolutionary idea that the future is somewhat predictable by a skilled individual, and not by seers.

The idea that the future is somewhat predictable is of course a necessary precondition to the development of a theory of probability. This is why the idea of measuring odds is a relatively recent development too. It was only in 1654 that a gambler's dispute on the profitability of betting on a given dice outcome led two famous French mathematicians, Blaise Pascal and Pierre de Fermat, to the creation of a mathematical theory of probability. What is surprising is that

[3] In Modern Greek, kairos denotes the weather, and this probably suggests a connection with the action of taking the sea at the opportune moment. A comparable semantic resemblance can be found in the old English maxim "time and tide wait for no man;" it is likely that time and tide were once the same word, this is still evident in the Dutch word for time: tijd.

[4] This dichotomy is likely to originate in Ancient Egypt, where a similar dichotomy is found between the concept of *neheh* (sequential time), and *djet* (time after the defunct has been checked by the judgment of the dead).

Figure 2: Kairos and Metanoia.

[5] Preston B. Cline. The Etymology of Risk. `https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.112.3920&rep=rep1&type=pdf`, 2004. Online; accessed 31 October 2020

[6] Alfred Crosby. *The measure of reality : quantification and Western society, 1250-1600*. Cambridge University Press, Cambridge England New York, NY, USA, 1997

gambling was widespread in the Greek and Roman world. Therefore it appears strange that, despite millennia of practice, no one managed to work out effective methods for computing the right odds for a given game. For instance, in the famous game of the astragali, the "Venus throw" — the one that earned the most — did not correspond to the outcome featuring the smallest odds; and some other equally probable throws earned less. Greeks and Romans played games of chance by rules that, by our standards, do not make any sense.[7]

[7] Peter Bernstein. *Against the gods : the remarkable story of risk*. John Wiley & Sons, New York, 1996

After focusing initially on games of chance, it soon become clear that probability theory could lead to profitable applications beyond this limited scope, e.g. to estimate mortality and to set the price of annuities.[8] The success of these applications continued well into the twentieth century, when applied probability played a key role during World War II. However, so-called *objective* probabilities were limited to very specific applications (e.g. games of chance, or laboratory experiments); in more complex settings, e.g. applications in economics or finance, probabilities were generally *subjective* and influenced by the amount of information to which the person computing such probabilities had access to. Moreover, in some cases, e.g. the crash of 1929, it was impossible to even conceive an event beforehand, let alone determine its probability.

[8] Edmond Halley. An estimate of the degrees of the mortality of mankind; drawn from curious tables of the births and funerals at the city of breslaw; with an attempt to ascertain the price of annuities upon lives. *Philosophical Transactions of the Royal Society of London*, 17 (196):596–610, December 1693

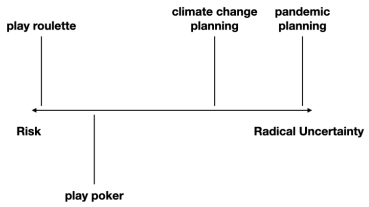

Figure 3: Risk vs Radical Uncertainty.

In response, Kay and King describe uncertainty as a continuum spanning between two extremes: *risk* and *radical uncertainty* (Fig. 3).

At one extreme, risk is that uncertainty that is measurable and amenable to being analysed by the tools offered by probability theory; this is the uncertainty paradigm of Ramsey[9] and Savage.[10] At the opposite extreme, radical uncertainty captures Donald Rumsfeld's *unknown unknowns*: "things that we don't know, and that we don't know that we don't know" (and hence we cannot measure); this is the uncertainty paradigm of Keynes[11] and Knight[12] — the so-called Knightian uncertainty.[13] According to Kay and King, in presence of radical uncertainty, sense-making trumps number-crunching.

> Decision-making under *radical uncertainty* requires a wide range of skills that can rarely be found in a single individual. [...] the use of the narrative and respect for diverse views can generate a better understanding of "what's going on here" than an overreliance on rigid econometric models. This does not mean that we should toss out the models entirely as they can give us a sense of direction and insight, but the sole reliance on them may not allow us to see the wood for the trees and understand the nature of the real issue.[14]

[9] Frank P. Ramsey. Truth and probability. In R. B. Braithwaite, editor, *The Foundations of Mathematics and other Logical Essays*, chapter 7, pages 156–198. McMaster University Archive for the History of Economic Thought, 1926

[10] Leonard J. Savage. *The Foundations of Statistics*. Wiley Publications in Statistics, 1954

[11] John Maynard Keynes. *A Treatise on Probability*. Dover Publications, 1921; and Frank H. Knight. *Risk, Uncertainty and Profit*. Houghton Mifflin Co, Boston, MA, 1921

[12] Frank H. Knight. *Risk, Uncertainty and Profit*. Houghton Mifflin Co, Boston, MA, 1921

[13] Yasuhiro Sakai. J. M. Keynes on probability versus F. H. Knight on uncertainty: reflections on the miracle year of 1921. *Evolutionary and Institutional Economics Review*, 13(1):1–21, May 2016

[14] J. A. Kay and M. A. King. *Radical uncertainty*. The Bridge Street Press, London, 2020

Not only it is necessary to rely on a wide range of skills to tackle radical uncertainty, but one must also recognise biases induced by our cultural background.[15] Despite the passage of time, the legacy of the scientific revolution is still strong in our day to day life, from education to language. Much of the western school curriculum devotes limited (if any at all) space to probability and uncertainty. Most mathematics and physics taught at school focus on deterministic models. This has an impact on how people approach the world. Peat illustrates how our language and culture affect our attitude towards uncertainty. By relying on ethnography, he shows how indigenous tribes display behaviours that appear irrational by our standards. For example, among the Blackfoot tribe, names of people, places, and objects are not fixed (i.e. certain). A name will change during a person lifetime to reflect particular deeds and attitudes. Personality is perceived as inherently fluid, not as a fixed concept.[16]

[15] F Peat. *From certainty to uncertainty : the story of science and ideas in the twentieth century*. Joseph Henry Press, Washington, D.C, 2002

[16] For us, featuring multiple personalities is a disorder; for them, it would mean missing out on the richness of life possibilities.

We conclude this section with two observations by Oreskes: first, more scientific research is unlikely to decrease uncertainty — in fact, it may increase it.[17] More information does not mean less uncertainty — in fact, it may lead to more uncertainty, and to conflicting positions among the scientists involved[18] — consider, for instance, the crisis and the debates generated by the discovery of the wave-particle duality. Second, mathematical decision support models are important and must feed into the scientific (or practical/political) debate; however, models alone are unlikely to resolve disputes, as the issues at stake are almost certainly not technical (e.g. moral, religious, aesthetic). For instance, Manski discusses how the *mathematically optimal* solution to administering a certain treatment to hospital patients may result culturally or ethically unacceptable to the population being treated.[19] In presence of high uncertainty about the effectiveness of the treatment, it may be in fact optimal to randomise treatments, to maximise population survival. However, such randomisation would be hard to explain/justify at an individual level, when one of the treatment is known to be (even just marginally) better than another. At the same time, administering the marginally better treatment to all patients would hinder learning and the discovery of better alternatives, and thus achieve a worse outcome for the population at large.

[17] Oreskes strongly rejects the paradigm of science as the process by which we "reduce" uncertainty. I share this viewpoint.

[18] Thomas S. Kuhn. *The structure of scientific revolutions*. University of Chicago Press, 1970

[19] Charles Manski. *Public policy in an uncertain world : analysis and decisions*. Harvard University Press, Cambridge, Mass, 2013. ISBN 978-0674066892

> There are known knowns. These are things we know that we know. There are known unknowns. That is to say, there are things that we know we don't know. But there are also unknown unknowns. These are things we don't know we don't know
>
> Former U.S. secretary of defence Donald Rumsfeld

> Avoid exposure to negative black swans, and seek exposure to positive.
>
> Taleb [2008]

I used to discuss these matters — and particularly the case of administering a certain treatment to hospital patients under high effectiveness uncertainty — with students years ago in the context of our undergraduate course "Introduction to Business." I would have never thought such an example would ever hit the headlines and generate so much debate across the whole country, as it eventually did when COVID-19 vaccines came out.

The Etymology of Risk

As Cline [2004] remarks, the current concept of risk is inherently unstable (Figure 4). This instability has clear operational costs, in the form of conceptual errors, as inconsistent lexicons are shared between academic and professional disciplines.

There have been several attempts to try and define the word "risk;" of which one of the most notable is probably summarised by the quoted text below.

> Many of you here remember that when our Society for Risk Analysis was brand new, one of the first things it did was to establish a committee to define the word "risk." This committee labored for 4 years and then gave up, saying in it's final report, that maybe it's better not to define risk. Let each author define it in his own way, only please each should explain clearly what that way is [Kaplan, 1997].

We shall next provide a bit of context to dispel a few misconception that persist about this term.

On the outskirts of London, in 1656, a Catholic attorney named Thomas Blount is unable to find work due to the ongoing Reformation.[20] As a result he decided to pursue his interest in lexicology and author the first mono-linguistic dictionary in the English Language. Within it, Mr. Blount would define a number of words including Risk.

> Risk: (Fr.) Peril, Jeopardy, Danger, Hazard, Chance.

In Blount's definition, there is a noticeable omission: the word Adventure. The net result is that the English language came to understand only half of the duality of risk: **the potential for loss.** From that point onward, issues that can play a role in the decision making process, such as courage, whimsy, serendipity, were labeled **irrational.** Consequently, there has been, and continues to be, paradoxes and exceptions that plague the English definition of the word risk [Cline, 2004].

In summary, it should be acknowledged that risk is, in fact, a neutral term. There are positive risks, as well as negative risks in any enterprise. The goal of Risk Management, is to do exactly what Taleb recommends while dealing with "black swans:" avoid exposure to negative risks, and seek exposure to positive ones. In this respect, Risk Management and Uncertainty Management do coincide.

We shall next survey established Risk Management practices commonly adopted in Project Management.

[20] Theo Bongaerts. *The correspondence of Thomas Blount (1618-1679), a recusant antiquary : his letters to Anthony Wood and other restoration antiquaries*. APA-Holland University Press, Amsterdam, 1978

Key event	Year	Risk measure used
Risk was fate or divine providence > prayer or sacrifice	Pre-1494	Gut feeling
Luca Pacioli posits coin tossing gamblers puzzle	1494	
Pascal, Fermat solve Pacioli puzzle, lay foundation for probability theory	1654	Computed probabilities
Graunt generates life table of births and deaths in London	1662	
Bernoulli states *"law of large numbers,"* providing basis for sampling from large populations	1711	Sample-based probabilities
De Moivre derives normal distribution as approximation to the binomial; Gauss and Laplace refine it	1738	
Bayes publishes treatise on how to update prior beliefs as new information is acquired	1763	
Insurance business develops, measures risk based on historical data	1800s	Expected loss
Bachelier examines stock and option prices on Paris exchanges, posits that prices follow a random walk	1900s	Price variance
Standard Statistics Bureau, Moody's and Fitch start rating corporate bonds using accounting information	1909–1915	Bond and Stock ratings
Markowitz lays statistical basis for portfolio diversification theory	1952	Variance added to portfolio
Sharpe, Lintner introduces riskless asset, show that combination with market portfolio are optimal for all investors > capital asset pricing model (CAPM)	1964	Market beta
Risk and return models based on alternatives to normal distribution	1960s	
Ross derives arbitrage pricing model; multiple market risk factors are derived from historical data	1976	Factor betas
Macroeconomics variables examined as potential market risk factors, leaning to multi-factor model	1986	Macroeconomic betas
Farma, French claim that market cap and book-to-price at better proxies for risk than beta or betas	1992	Proxies

Damodaran, Aswath. *Strategic Risk Taking: How Do We Measure Risk?* New York University. Web. 20 June 2015

Figure 4: A timeline in risk analysis and evolution of risk measures [Ata, 2016].

Introduction

This book originates as a collection of self-contained lectures. These lectures are divided into an introduction to inventory control, which outlines the foundations of inventory systems; followed by three chapters on deterministic inventory control, demand forecasting, and stochastic inventory control.

Beside Inventory, the title of the book refers to Analytics. This is nowadays a concept that has been inflated with a plethora of meanings, so that it becomes difficult to understand exactly what each of us means when we refer to it. The Cambridge Dictionary[1] defines Analytics as "a process in which a computer examines information using mathematical methods in order to find useful patterns." However, this appears to be quite a restrictive definition for our purposes.

[1] *Cambridge Dictionary*, https://dictionary.cambridge.org/dictionary/english/analytics?q=Analytics

To better understand the nature of Analytics, it is useful to observe that Analytics is often broken down into three parts: descriptive, predictive, and prescriptive. Descriptive Analytics is concerned with answering the question: "what happened?" Predictive Analytics is concerned with answering the question: "what will happen?" Prescriptive Analytics is concerned with answering the question: "how can we make it happen?" These are clearly complex questions that cannot be answered by mere *number crunching* on a computer: to answer these questions a decision maker must leverage soft as well as hard skills.

Fig. 1 This allegorical woodcut shows Rhetorica enthroned between Prudentia and Inventio; Willem Silvius, Antwerp, 1561 (Image by Anonymous, Wikimedia, public domain).

Many tend to think that the Analytics phenomenon is a recent development related to widespread availability of computing power. However, in his work "De Inventione," the Roman philosopher Cicero states that "there are three parts to Prudence: Memory, Intelligence, and Foresight." It is clear that Memory is the skill required to answer the question "what happened?"; Foresight, that required to answer the question "what will happen?"; and Intelligence, that required to answer the question "how can we make it happen?" It appears then that Analytics is just a contemporary rebranding of an art that has been known for millenia. *Prudentia* is the ability to govern and discipline oneself by the use of reason. *Inventio* is the central canon of rhetoric, a method devoted to systematic search for arguments. Incidentally, *inventio* also means inventory. In fact, when a new argument is found, it is *invented*, in the sense of "added to the inventory" of arguments. *Prudentia* and *Inventio* are the foundations upon which the art of Rhetoric stands (Fig. 1).

https://doi.org/10.11647/OBP.0252.06

It must not surprise us then that Analytics plays a prominent role in inventory management. Inventory management finds its roots into the practice of late medieval and early Renaissance merchants.[2] The invention of double-entry bookkeeping (alla Veneziana) is typically attributed to Frà Luca Pacioli (c. 1447 – 19 June 1517). Pacioli leveraged Johannes Gutenberg's new technology to disseminate and popularise accounting practices that had been in use among Venetian merchants for a long time. However, Pacioli did not simply disseminate existing practices, he reinterpreted these practices within the framework of Cicero's rethoric.[3] In "De Inventione," Cicero explains that there are five canons, or tenets, of Rhetoric: *Inventio* (invention), *Dispositio* (arrangement), *Elocutio* (style), *Memoria* (memory), and *Pronuntiatio* (delivery).

Pacioli's "Tractatus de computis et scripturis" (1494, Fig. 2), is divided into two main sections: (i) the Inventory, and (ii) the Disposition — the influence of Cicero's work is apparent. Pacioli writes: "In order to conduct a business properly a person must: possess sufficient capital or credit, be a good accountant and bookkeeper, and possess a proper bookkeeping system." In "the Inventory," Pacioli writes "The merchant must prepare a list of his inventory. Items that are most valuable and easier to lose should be listed first. [...] The inventory should be carried out and completed in a single day. [...] The inventory is to include the day that the inventory was taken, the place, and the name of the owner."[4] In contemporary terms, Pacioli describes a so-called "physical inventory," the process by which a business physically reviews its entire inventory — as opposed to so-called "cycle counts," which focus on specific subsets of items. In "the Disposition," Pacioli describes the necessary books and rules to implement double-entry bookkeeping.[5]

Pacioli's work represents a quantum leap in the realm of *descriptive inventory analytics*, a discipline that would evolve into a fundamental part of inventory management. However, no progress was made in the realm of *predictive* and *prescriptive inventory analytics* until late 1800, when Edgeworth,[6] in his "Mathematical Theory of Banking," used the central limit theorem to determine cash reserves needed to satisfy random withdrawals from depositors, thus embedding a *predictive* probabilistic model within a *prescriptive* mathematical model to support inventory control decisions.

From these early results, over the past 150 years, inventory control has evolved into an independent discipline. The aim of this book is to provide an introduction to this discipline.

After introducing the foundations of inventory systems, in chapter "Deterministic Inventory Control" we survey *prescriptive analytics* models for deterministic inventory control, in chapter "Demand Forecasting" we discuss *predictive analytics* techniques for demand forecasting in inventory control, which originate in the realm of time series analysis and forecasting. Finally, in chapters "Stochastic Inventory Control" and "Multi-echelon Inventory Systems" we survey *prescriptive analytics* models for stochastic inventory control.

[2] Alfred Crosby. *The measure of reality: quantification and Western society, 1250-1600*. Cambridge Univ. Pr., 1997.

[3] Paolo Quattrone. Books to be practiced: Memory, the power of the visual, and the success of accounting. *Accounting, Organizations and Society*, 34(1):85–118, 2009.

Fig. 2 Dedication page of Pacioli's "Tractatus de computis et scripturis;" printed by Paganino de Paganini, Venice, 1494 (courtesy of Wellcome Collection).

[4] William A. Bernstein. Luca pacioli the father of accounting. In *The Air Force Comptroller*, volume 10(2) of *Air Force recurring publication 170-2*, pages 44–45. Office of the Comptroller, United States Air Force, 1976.

[5] Paolo Quattrone. Governing social orders, unfolding rationality, and Jesuit accounting practices. *Administrative Science Quarterly*, 60(3):411–445, 2015.

[6] Francis Y. Edgeworth. The mathematical theory of banking. *Journal of the Royal Statistical Society*, 51(1):113–127, 1888.