# La Nova Scientia: Rewriting the History of Operational Research

Roberto Rossi

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The accepted lore is that Operational Research traces its roots back to the First and Second World Wars, when scientific research was used to improve military operations. In this essay we provide a different perspective on the origins of Operational Research by arguing that these are deeply intertwined with the impressive technological advances in Western Europe between the fifteenth and the sixteenth centuries.

Subjects: **History and Overview (math.HO)**
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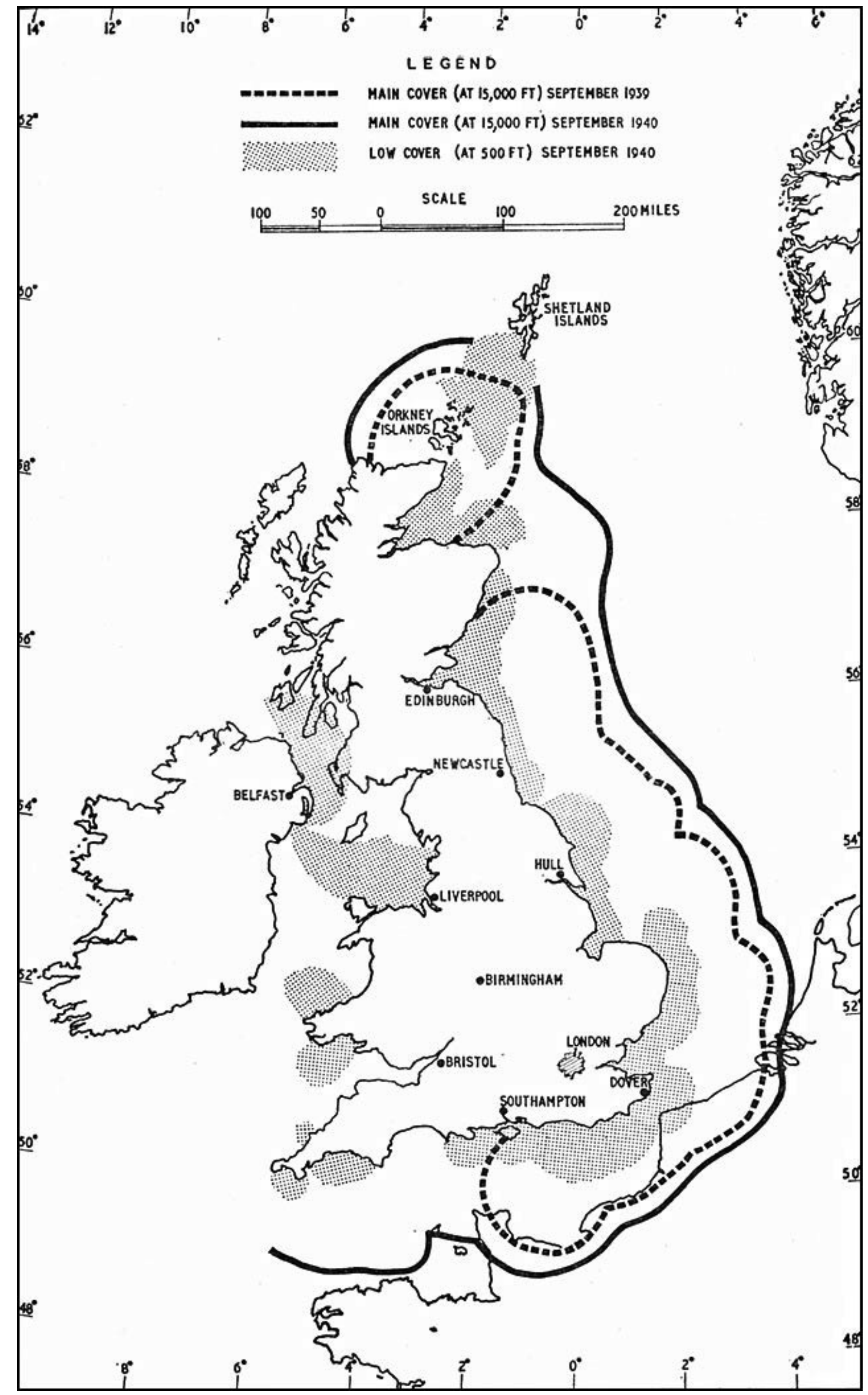
“Operational Research (OR) is the application of the methods of science to complex problems arising in the direction and management of large systems of men, machines, materials, and money in industry, business, and defence.”

– *“Operational Research in War and Peace: The British Experience from the 1930s to 1970,” Kirby (2003)*

The accepted lore is that OR traces its roots back to the First and Second World Wars, when **scientific research was used to improve military operations**.

C. M. Molinero. Operational research: From war to community. Socio-Economic Planning Sciences, 26(3):203–212, July 1992.

E. P. Rau. Combat science: the emergence of operational research in world war II. Endeavour, 29(4):156–161, Dec. 2005.

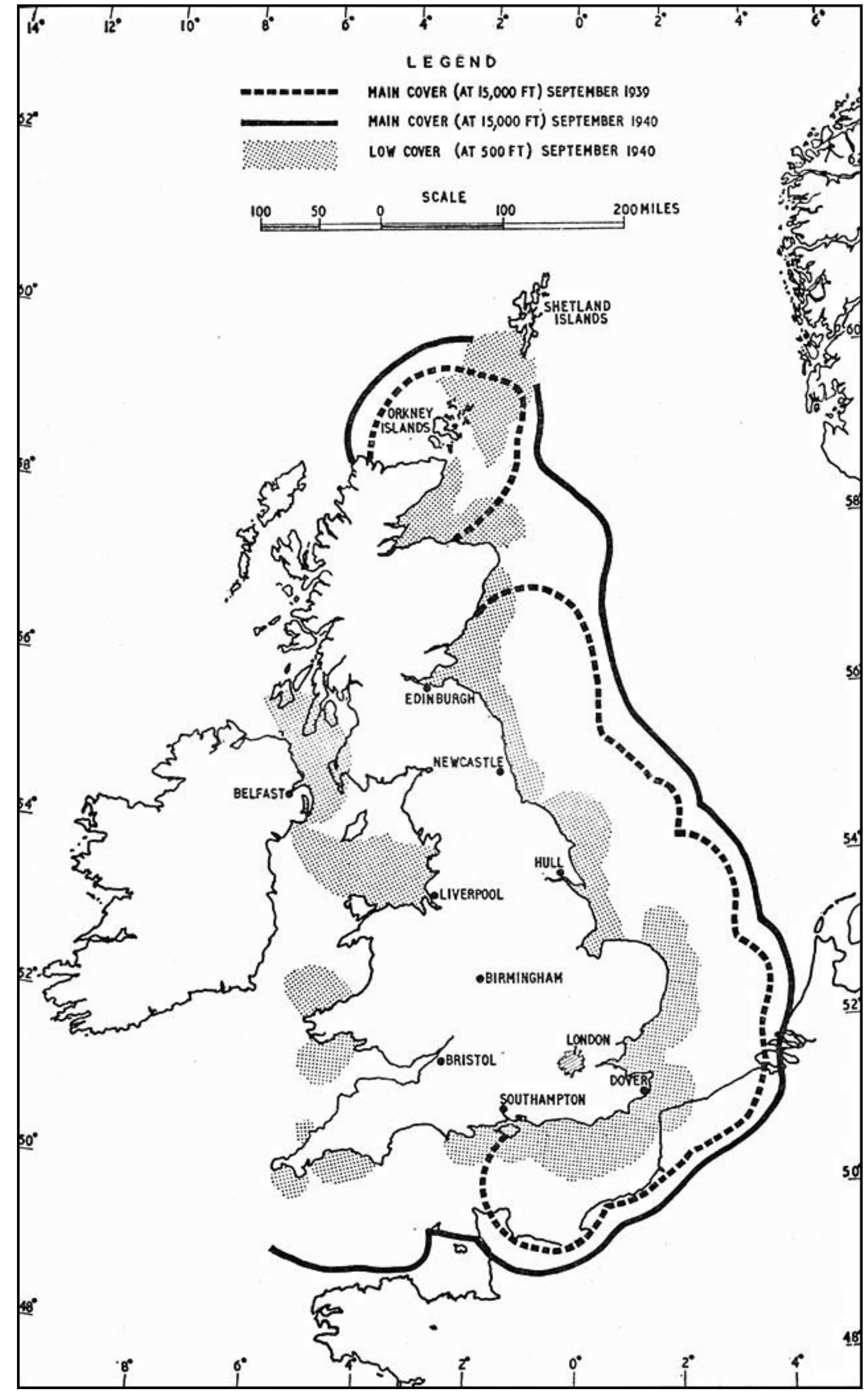



**UK radar coverage 1939–1940**

Optimal **positioning of radar bases** along the UK coastline during the Battle of Britain.

Military **resource allocation problems** in the context of the trade-off between aerial antisubmarine-warfare and strategic bombing operations in the Continent.

**Cryptanalytic techniques** such as Banburismus and Scritchmus developed at Bletchley Park.




**UK radar coverage 1939–1940**

"Whilst the discipline and practice of OR originated in the later 1930 and came to fruition during the Second World War, the history of warfare is punctuated by attempts to apply some elements of quantitative analysis to understanding the causes of victory and defeat…"

*– "Operational Research in War and Peace: The British Experience from the 1930s to 1970," Kirby (2003)*

**“…the first overt attempt to apply algebra and numbers to the battlefield occurred in later sixteenth century.”**

*– “Operational Research in War and Peace: The British Experience from the 1930s to 1970,” Kirby (2003)*

“…as soon as the first rank has fired together, […] they will march to the back. The second rank, either marching forward or standing still, [will next] fire together [and] then march to the back. After that, the third and following ranks will do the same. Thus before the last ranks have fired, the first will have reloaded.”

*–Letter from Louis to Maurice (Counts of Nassau, 1585 - 1625)*

However, Kirby defines Maurice an “unconscious practitioners” of OR.

A natural question then arises…

*Were military practitioners of the sixteenth century simply following common sense, or did some of them follow a principled, scientific approach akin to the one in use today?*

What would happen if we read their research works today?

The fifteenth and sixteenth centuries saw impressive technological advances.

“Painting, cartography and ballistic do not strike us as cutting-edge sciences, but once they were.”

*–Valleriani, “Metallurgy, Ballistics and Epistemic Instruments” (2013)*

# Painting

**Il “terremoto di Efeso,” abside chiesa di San’t Agostino, Rimini, Italy (c.a 1350)**

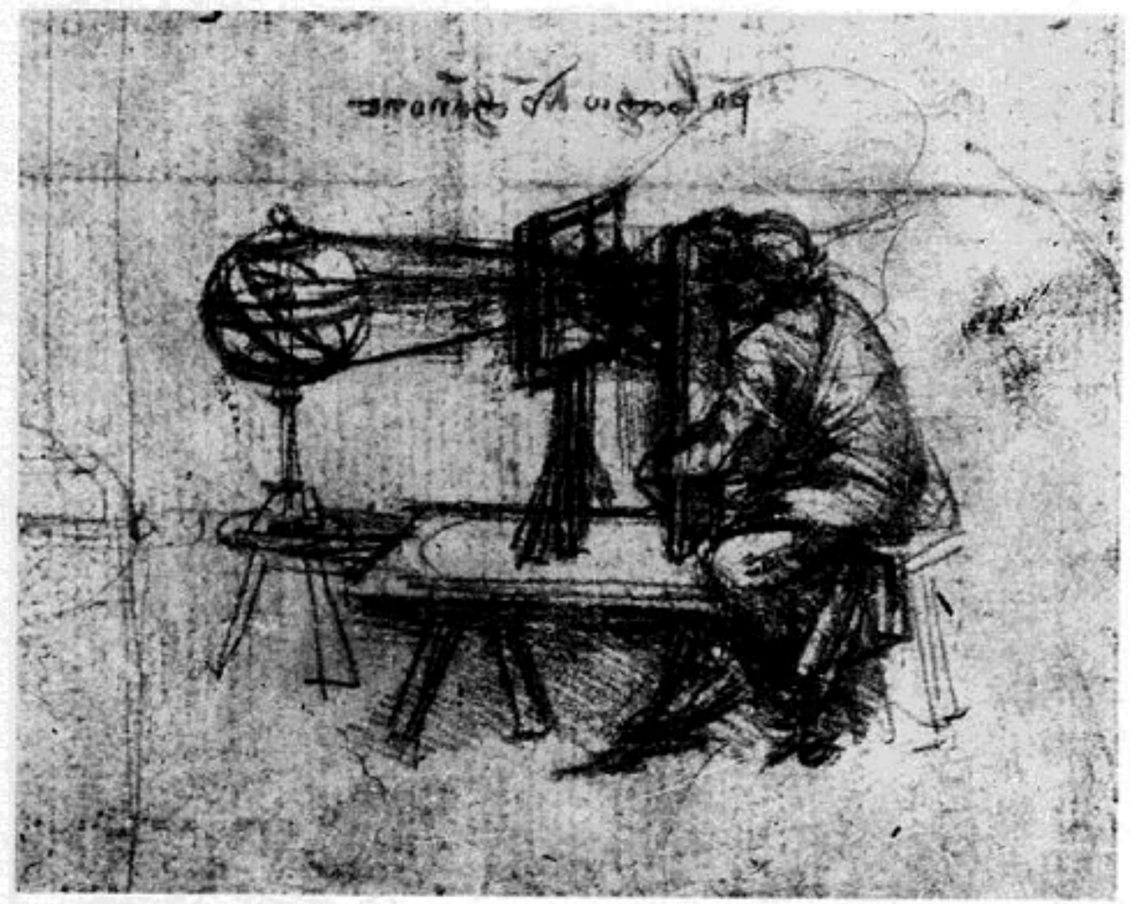
# Painting

**La città ideale, autore sconosciuto (1480-90)**

# Painting

As Brunelleschi (1377–1446) made no written record of his **perspective** findings, it remained for L. B. Alberti to be the first to put the theory into writing, in his treatise on painting, “Della Pittura” (1435).

Alberti described how an artist could get a correct view of a scene by observing it through a thin veil, or velo.



327. Leonardo da Vinci, *Draughtsman using a Transparent Plane to Draw an Armillary Sphere*, c.1510, Milan, Biblioteca Ambrosiana, Codice Atlantico lra (new 5).

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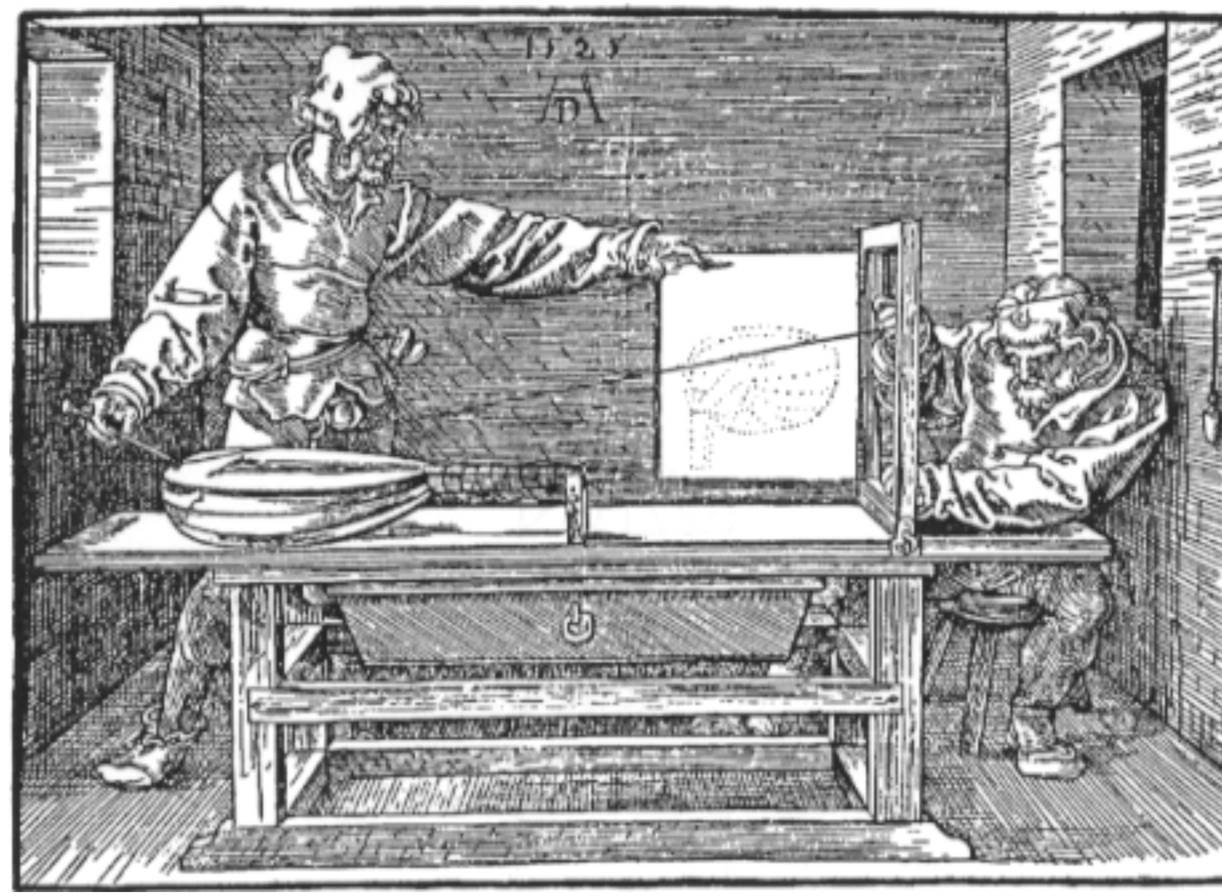
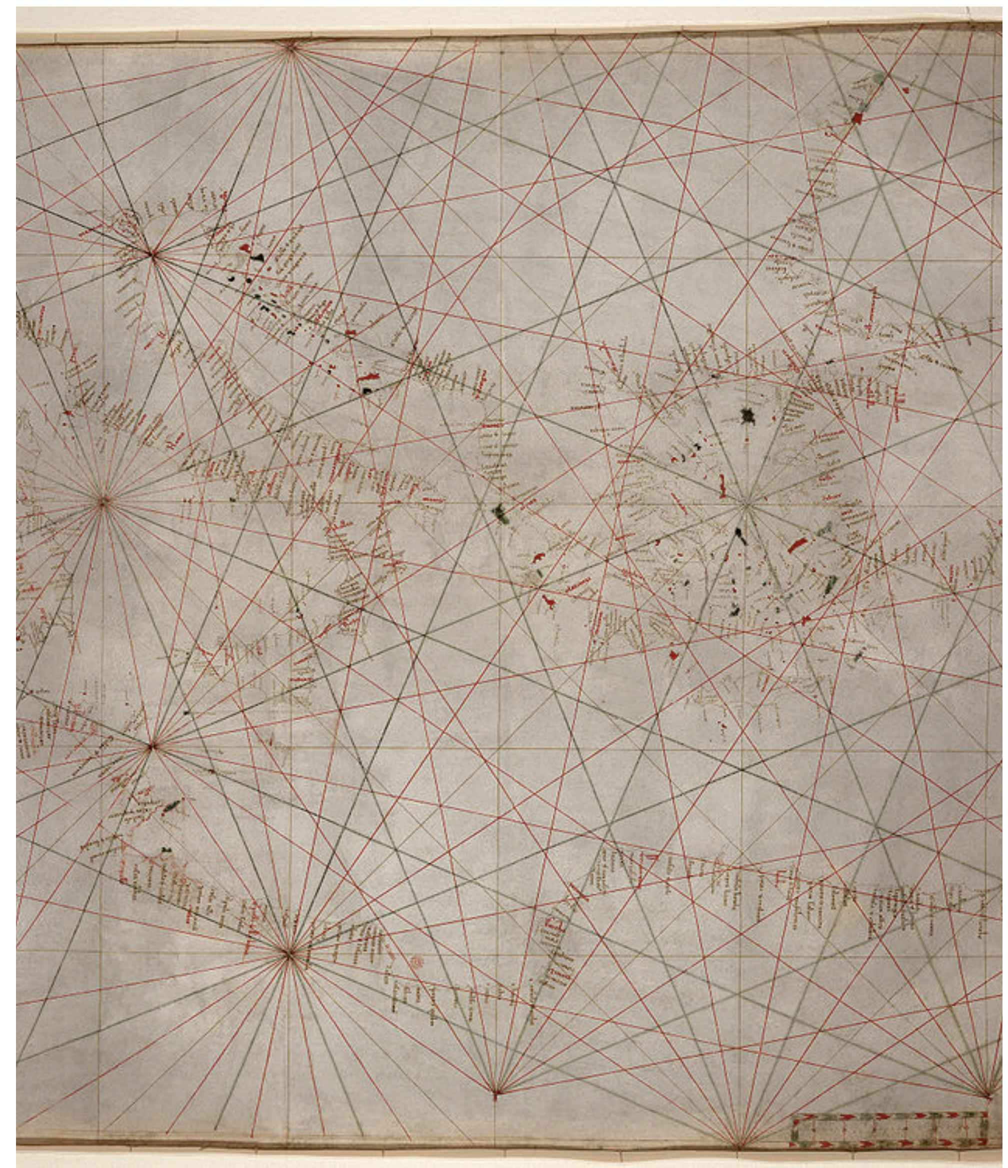


**Albrecht Durer's Perspective Machines (1525)**

# Cartography

The Carta Pisana is the oldest surviving “portolano,” i.e. nautical chart.

It is not simply a map but a document showing accurate navigational directions.



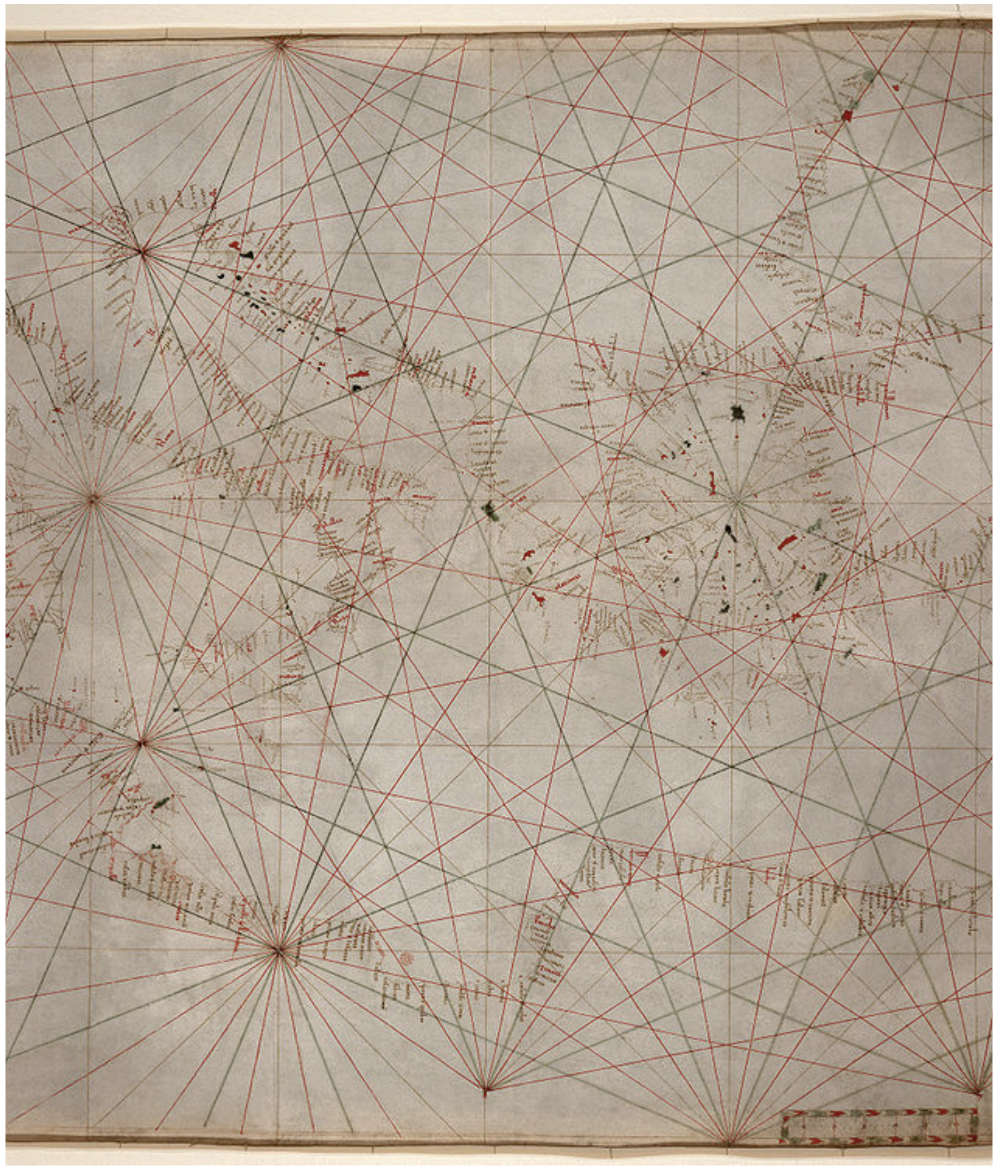
**Carta Pisana portolan chart (1275–1300)**

# Cartography

The Carta Pisana is the oldest surviving “portolano,” i.e. nautical chart.

It is not simply a map but a document showing accurate navigational directions.

Best used in conjunction with a **compass**!

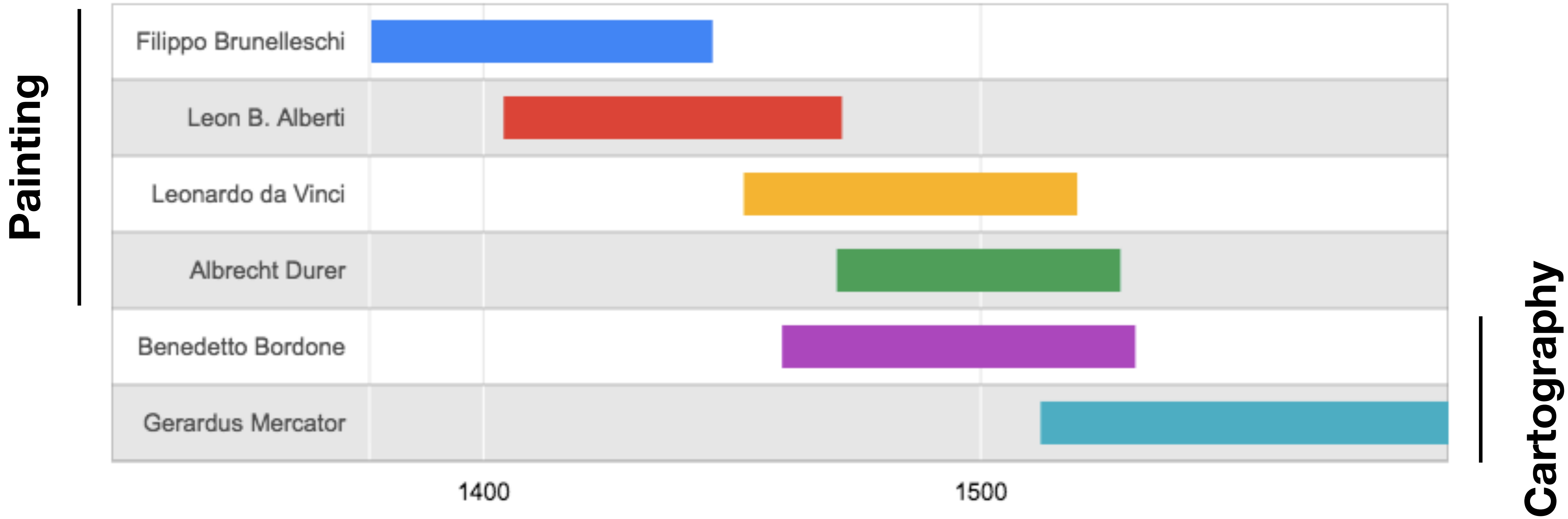


**Carta Pisana portolan chart (1275–1300)**

# Cartography

Mercator's **projection** turns sailing courses of a constant bearing into straight lines on the map thus greatly facilitating navigation.

**Mercator's 1569 world map**


Painting
Filippo Brunelleschi
Leon B. Alberti
Leonardo da Vinci
Albrecht Durer
Benedetto Bordone
Gerardus Mercator
1400
1500
Cartography

# Ballistics

Valleriani (2013) investigated how and why a new **theoretical ballistic** had emerged in the sixteenth century.

Equilibrium between attack and defence strategies that had endured for centuries has been destroyed by introduction of **gunpowder/heavy artillery**.

The emergence of new **theoretical knowledge** should be intended as a consequence of an advanced and challenging technological context.

**Luis Collado. Prattica manuale dell'artiglieria. Milano, 1606, p. 110**

Both Kirby (2003) and Valleriani (2013) date early attempts to apply algebra and numbers to the battlefield in the **late sixteenth century**.

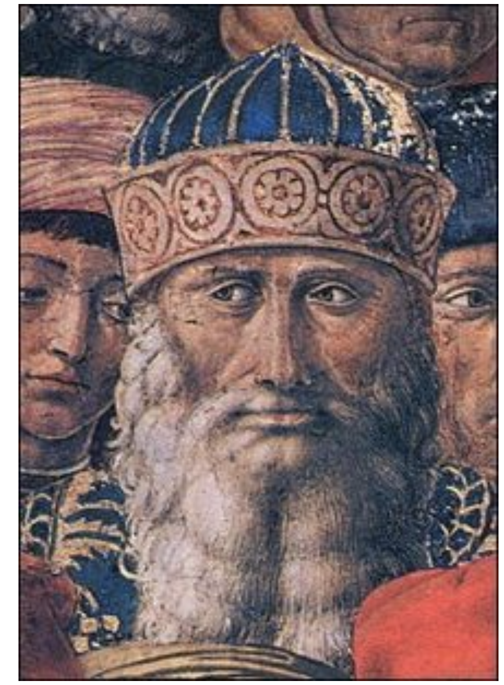
*But how did it all started and what are the essential ingredients (if any) to the OR recipe?*

The **decline** and collapse of the Byzantine empire heightened contact between its scholars and those of the west and brought an influx of **Neoplatonic** scholars.

**Constantine XI Dragases Palaiologos, (8 February 1405 – 29 May 1453) was the last reigning Byzantine Emperor**

Georgius Gemistus (c. 1355–1452/1454) **reintroduced Plato’s thoughts** to Western Europe during the 1438–1439 Council of Florence.

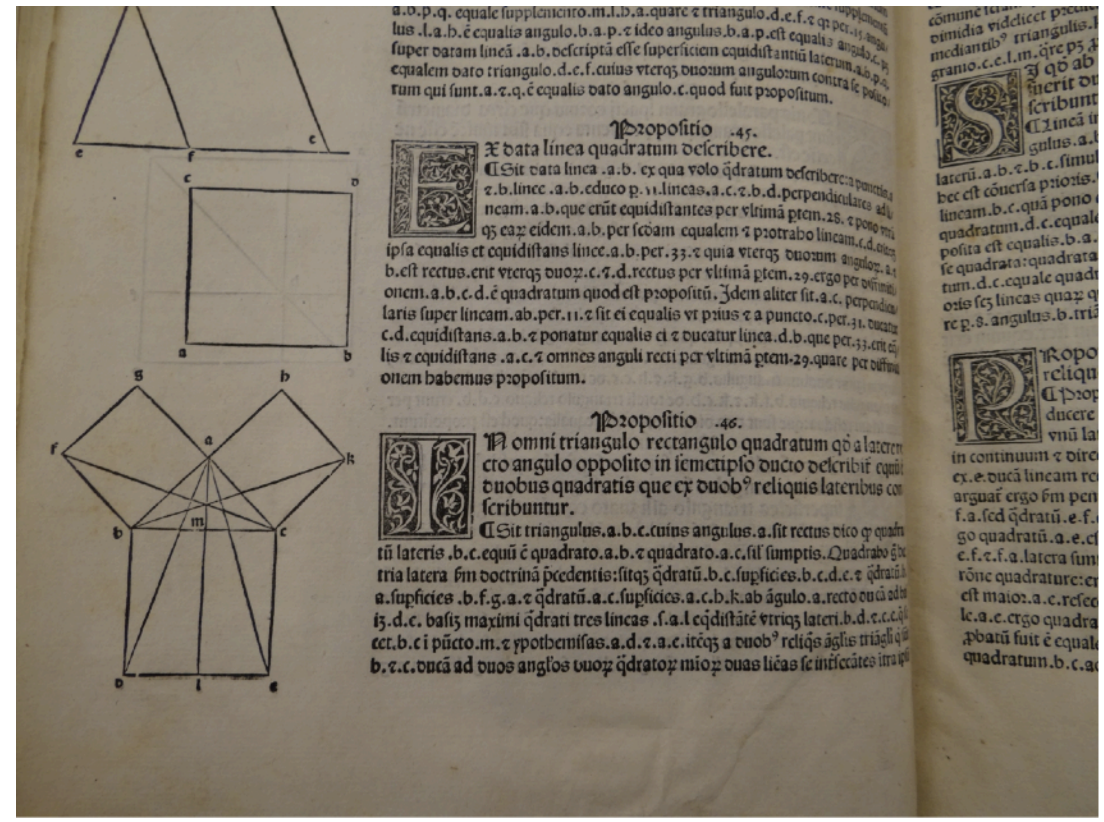
He founded a new Platonic Academy that focused on the **translation** of **Plato** into Latin.



**Georgius Gemistus (c. 1355–1452/1454)**

**Gemistus’ tumb, Rimini, Italy**

As a consequence of the Renaissance revival of Greek mathematics and of the rational tradition of Greek science, we find a **proliferation** of **studies** and **translations** of **greek works**, including **Euclid**.

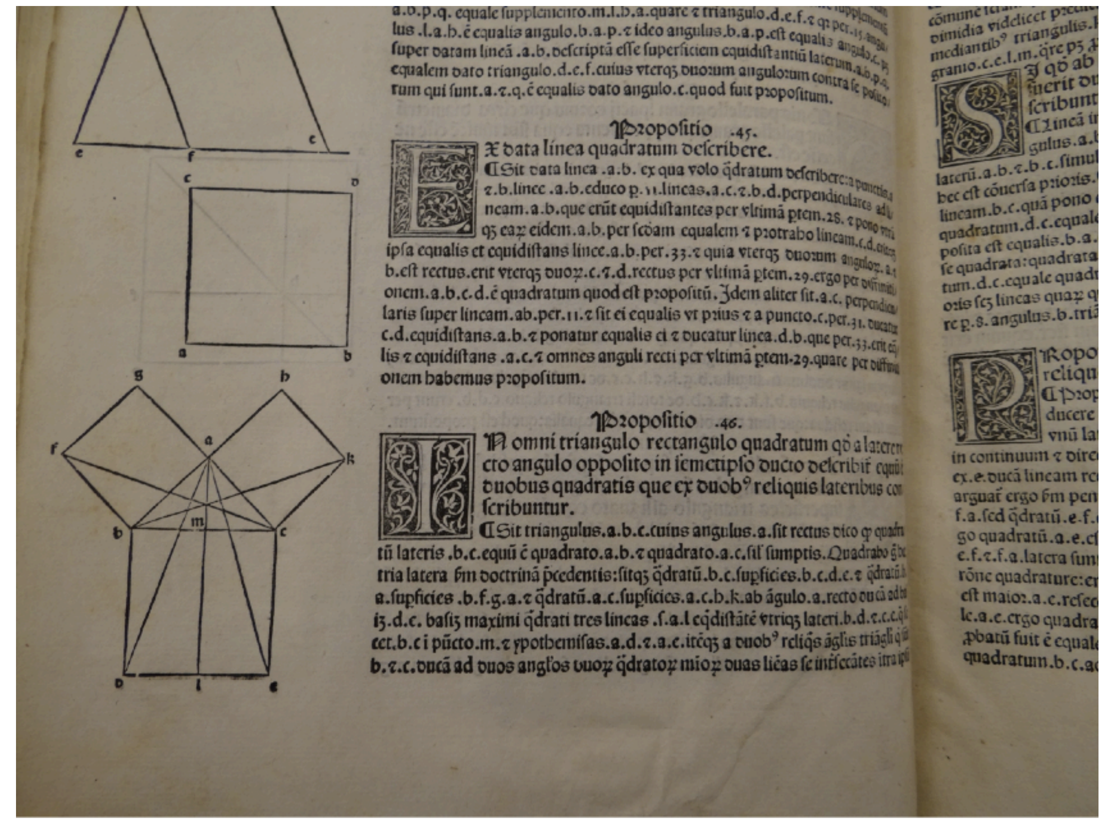



**Euclid's Elements, in Latin, published by Erhard Ratdolt on May 25, 1482, in Venice**

Euclid’s **Elements** was translated into Arabic in the ninth century.

Muslim mathematicians then combined geometry with Hindus arithmetic and algebra and developed new advances.

In the twelfth century the work was **translated** into **Latin**, making it more accessible to European scholars.

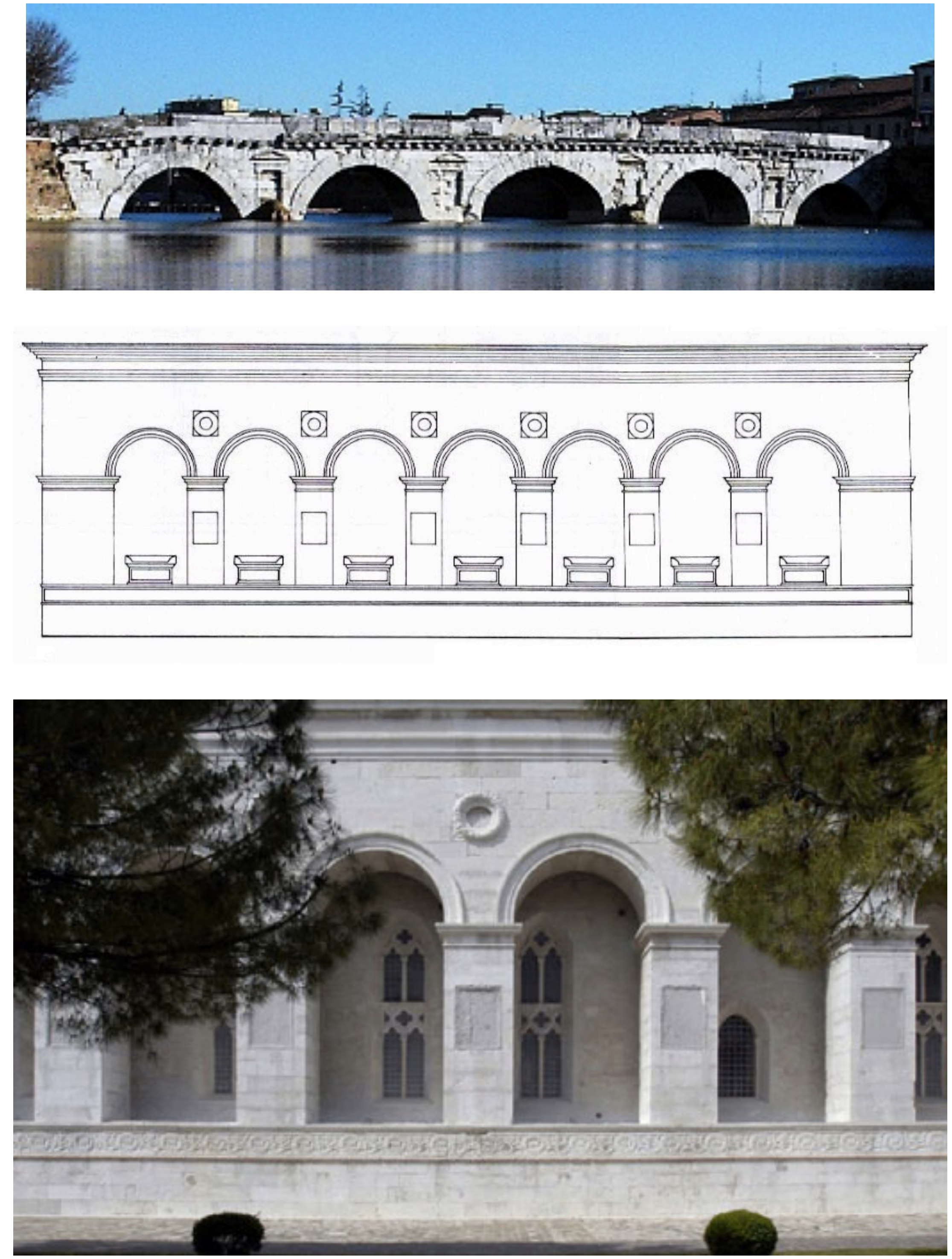


**Euclid’s Elements, in Latin, published by Erhard Ratdolt on May 25, 1482, in Venice**

**Neoplatonism** influence was strong at the time of Leon Battista Alberti (1404–1472).

**Mathematics**, as a discipline, tended to be framed in a Neoplatonic context.

K. Williams, L. March, and S. R. Wassell, editors. The Mathematical Works of Leon Battista Alberti. Birkhauser, Sept. 2010. ISBN 3034604734.

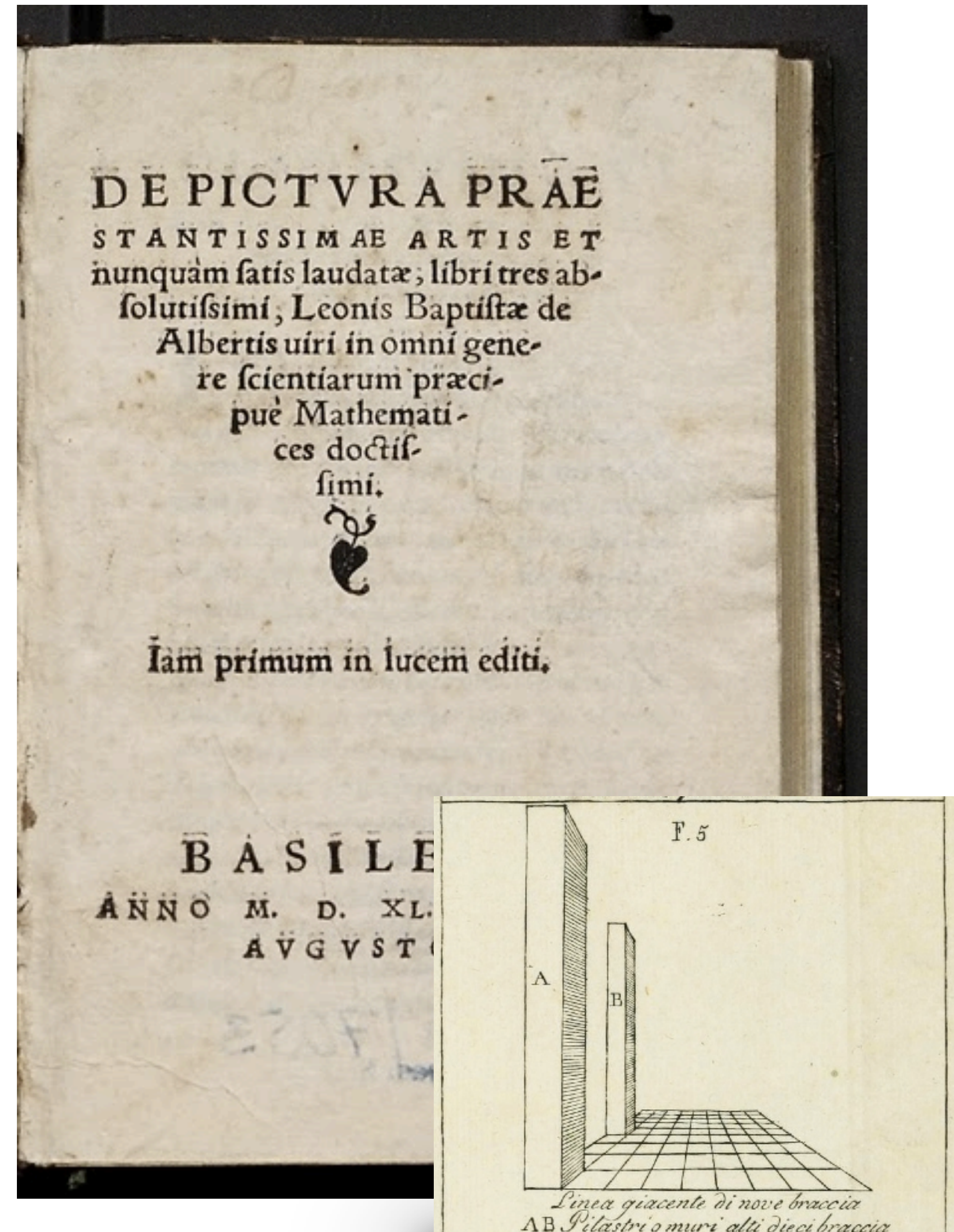


**L. B. Alberti, Tempio Malatestiano (1450), Rimini, Italy**

In his treatise “On Painting” Leon Battista Alberti wrote:

> “*Mathematicians measure the shapes and forms of things in the mind alone and divorced entirely from matter*.”

Reinassance Platonists had been interested in **number mysticism** rather than real mathematics (Yates, 2002).



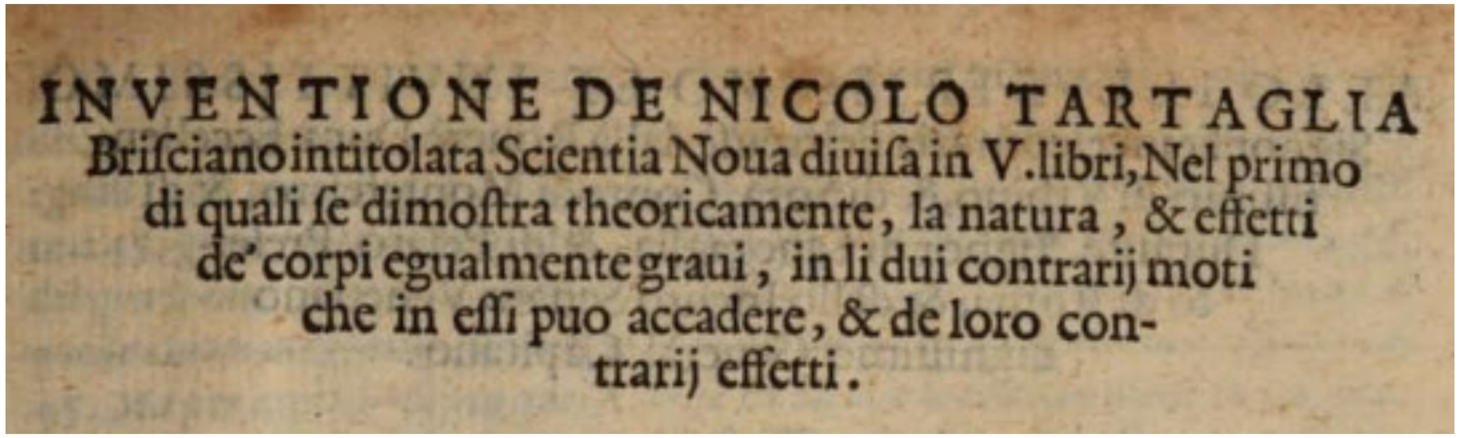

L. B. Alberti, De Pictura (1435)

A key ingredient of the OR recipe was missing:

*the connection between*
*theoretical models and practical applications*

The **development of this connection** is what motivated the studies discussed in Tartaglia's

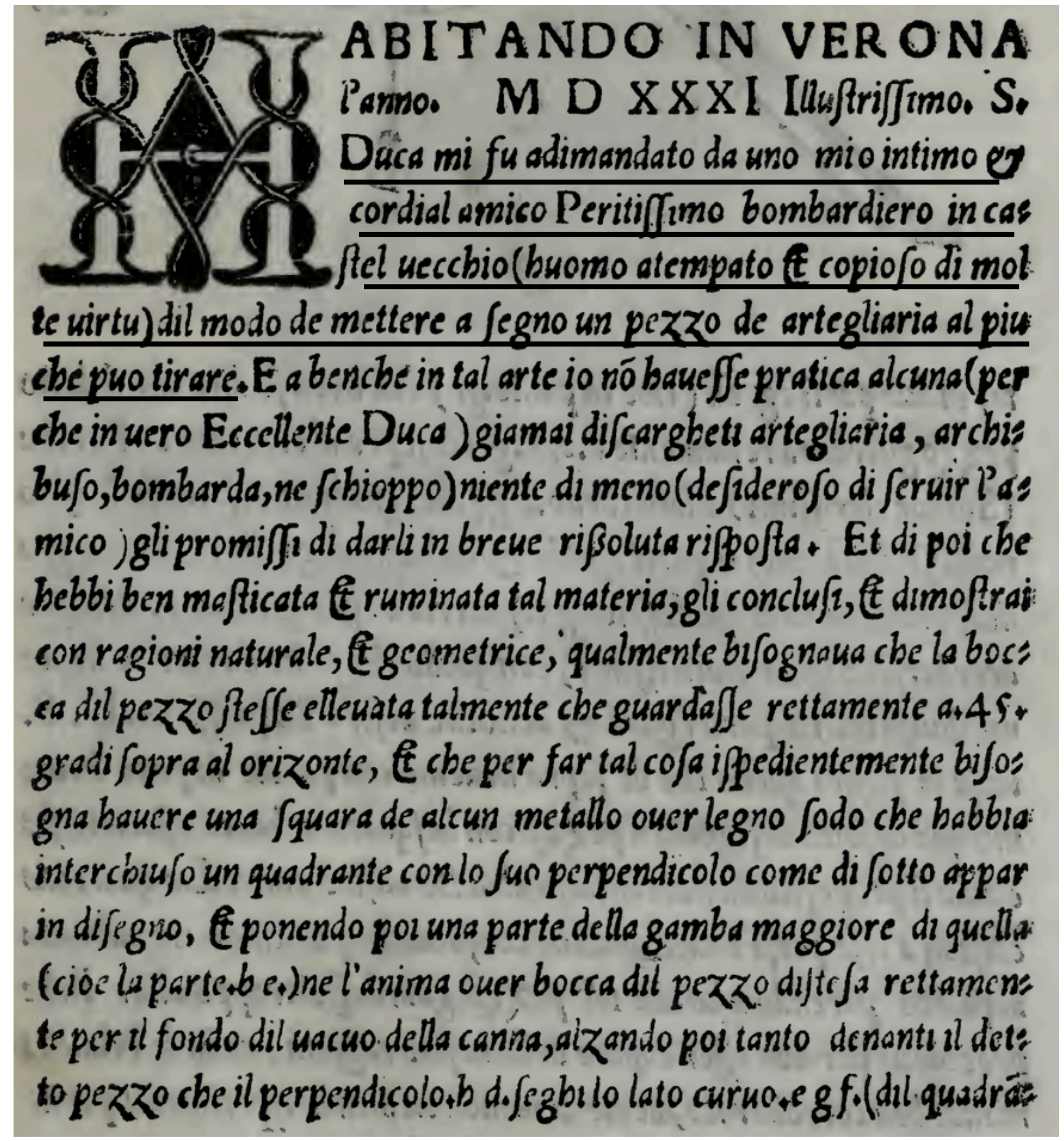
"La Nova Scientia" (1537)



INVENTIONE DE NICOLO TARTAGLIA
Briſciano intitolata Scientia Noua diuiſa in V.libri, Nel primo
di quali ſe dimoſtra theoricamente, la natura, & effetti
de' corpi egualmente graui, in li dui contrarij moti
che in eſſi puo accadere, & de loro con-
trarij effetti.

In 1532, while he was living in Verona, a friend who was a bombardier asked Tartaglia:

*At which angle the barrel of a cannon should be elevated to achieve the longest possible shot?*


HABITANDO IN VERONA l'anno. M D XXXI Illuſtriſſimo. S. Duca mi fu adimandato da uno mio intimo & cordial amico Peritiſſimo bombardiero in caſtel uecchio (huomo atempato & copioſo di molte uirtu) dil modo de mettere a ſegno un pezzo de artegliaria al piu che puo tirare. E a benche in tal arte io nõ haueſſe pratica alcuna (perche in uero Eccellente Duca) giamai diſcargheti artegliaria, archibuſo, bombarda, ne ſchioppo) niente di meno (deſideroſo di ſeruir l'amico) gli promiſſi di darli in breue riſſoluta riſpoſta. Et di poi che hebbi ben maſticata & ruminata tal materia, gli concluſi, & dimoſtrai con ragioni naturale, & geometrice, qualmente biſognaua che la bocca dil pezzo ſteſſe elleuata talmente che guardaſſe rettamente a. 45. gradi ſopra al orizonte, & che per far tal coſa iſpedientemente biſogna hauere una ſquara de alcun metallo ouer legno ſodo che habbia interchiuſo un quadrante con lo ſuo perpendicolo come di ſotto appar in diſegno, & ponendo poi una parte della gamba maggiore di quella (cioe la parte. b e.) ne l'anima ouer bocca dil pezzo diſteſa rettamente per il fondo dil uacuo della canna, alzando poi tanto denanti il detto pezzo che il perpendicolo. b d. ſeghi lo lato curuo. e g f. (dil quadrã

**Niccolò Tartaglia, “La Nova Scientia” (1537)**

Tartaglia did not have expertise in specialised areas connected to military activity.

However, having made some calculations, he was able to establish **on geometric and algebraic grounds** that the maximum range would be achieved if the barrel of a cannon were raised at an angle of 45 degrees above the line of horizon.

HABITANDO IN VERONA
l'anno. M D XXXI Illustrissimo. S.
Duca mi fu adimandato da uno mio intimo &
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**Niccolò Tartaglia, “La Nova Scientia” (1537)**

By answering this question, Tartaglia **consciously engaged** in an enquiry that today we would have no problem in labelling as OR.

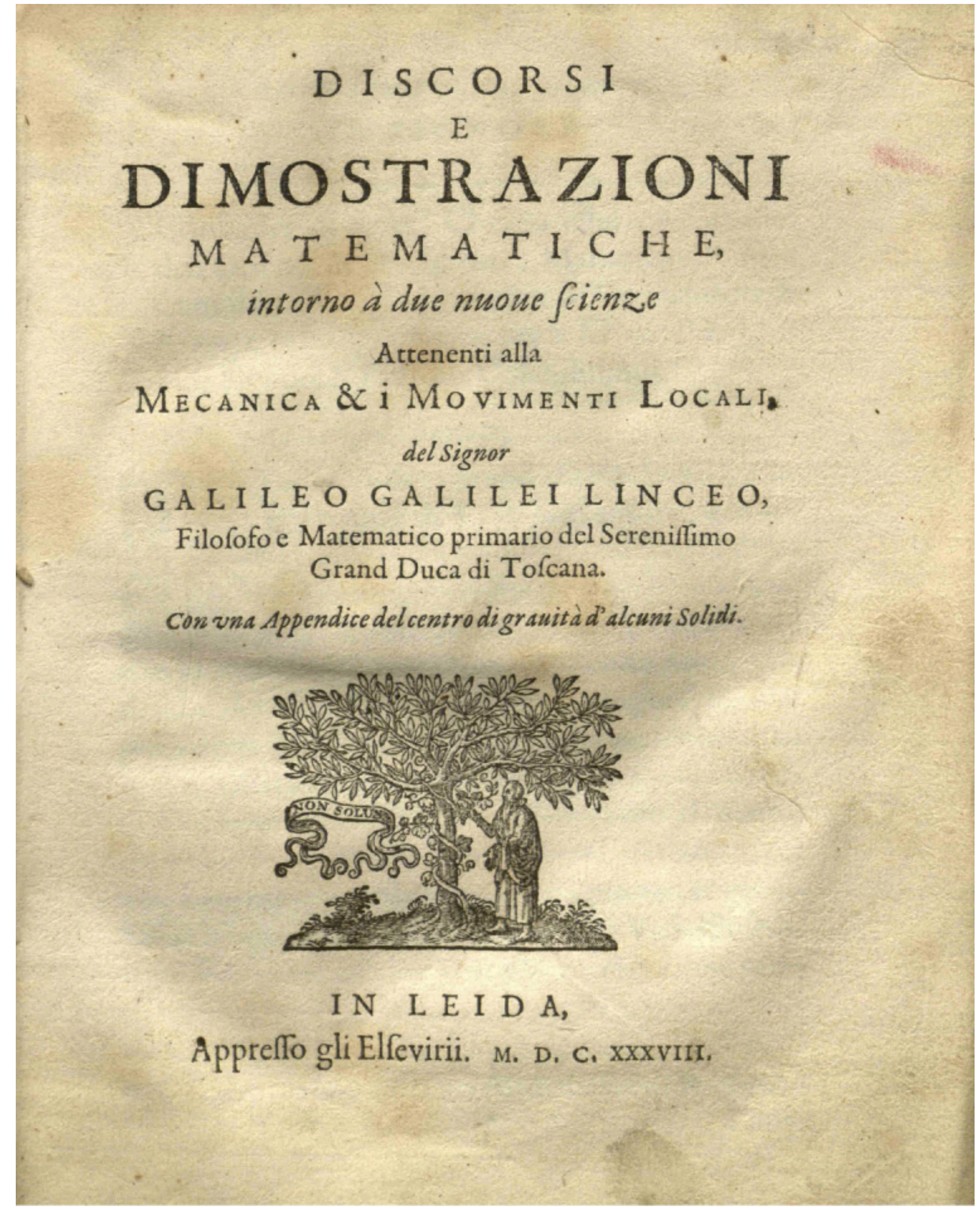
In Tartaglia’s time there had been a proliferation of mathematical treatises on topics such as **algebra** and **combinatorics**.

Both Ramon Lull and Gerolamo Cardano wrote an “Ars Magna.”


**Cardano's Ars Magna ("The Great Art") published in 1545**

What made Tartaglia’s work new (hence the title “Nova Scientia”) was the application of abstract mathematical models **to achieve practical outcomes**.

This was a revolutionary step that paved the way and inspired - as it is clear from the title of his treatise “Discorsi intorno a due **nuove scienze**” - Galileo’s works.



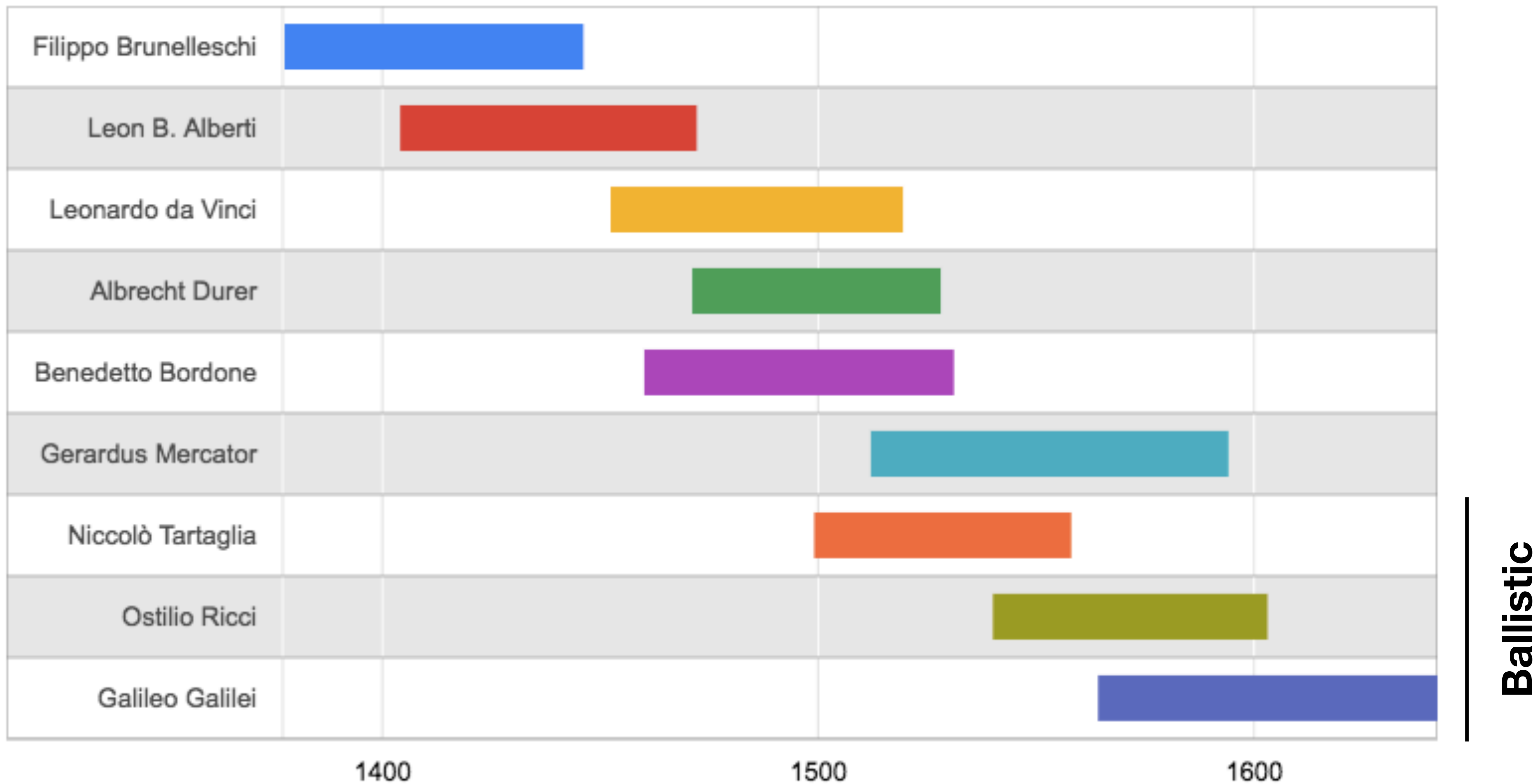

**Galileo Galilei: Discorsi e Dimostrazioni Matematiche Intorno a Due Nuove Scienze, 1638**

In the **frontispiece** of his work Tartaglia provides a motto stating that that the **mathematical disciplines** are seen as **the only method to understand “the reasons of things” and that “is open to every one;”**

This sets a precedent to Galileo’s Book of Nature and represents a clear cut from the Hermetic and Neoplatonic traditions, in which **knowledge is esoteric**.

**"Aurum probatur igni, et ingenium mathematicis"**


**Disciplinae mathematicae loquuntur: «Qui cupitis rerum varias cognoscere causas, discite nos: cunctis hac patet una via»"**



Filippo Brunelleschi
Leon B. Alberti
Leonardo da Vinci
Albrecht Durer
Benedetto Bordone
Gerardus Mercator
Niccolò Tartaglia
Ostilio Ricci
Galileo Galilei
1400
1500
1600
Ballistic

However, the connection between **theoretical models** and **applications** requires a further ingredient not yet discussed…

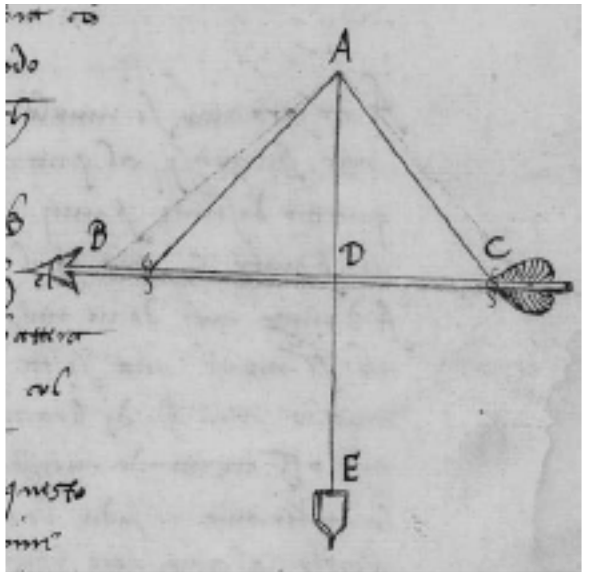
However, the connection between **theoretical models** and **applications** requires a further ingredient not yet discussed:

**measurement**.

Influenced by Euclid’s work, Leon Battista Alberti in his work “Ex Ludis rerum mathematicarum” discussed **applications of trigonometry to surveying** (Williams et al., 2010, 2013).

The methods discussed relied on instruments called **equilibra**.

Equilibra are a simple **extension of the plumb line** and could be used to measure angles in everyday activities.



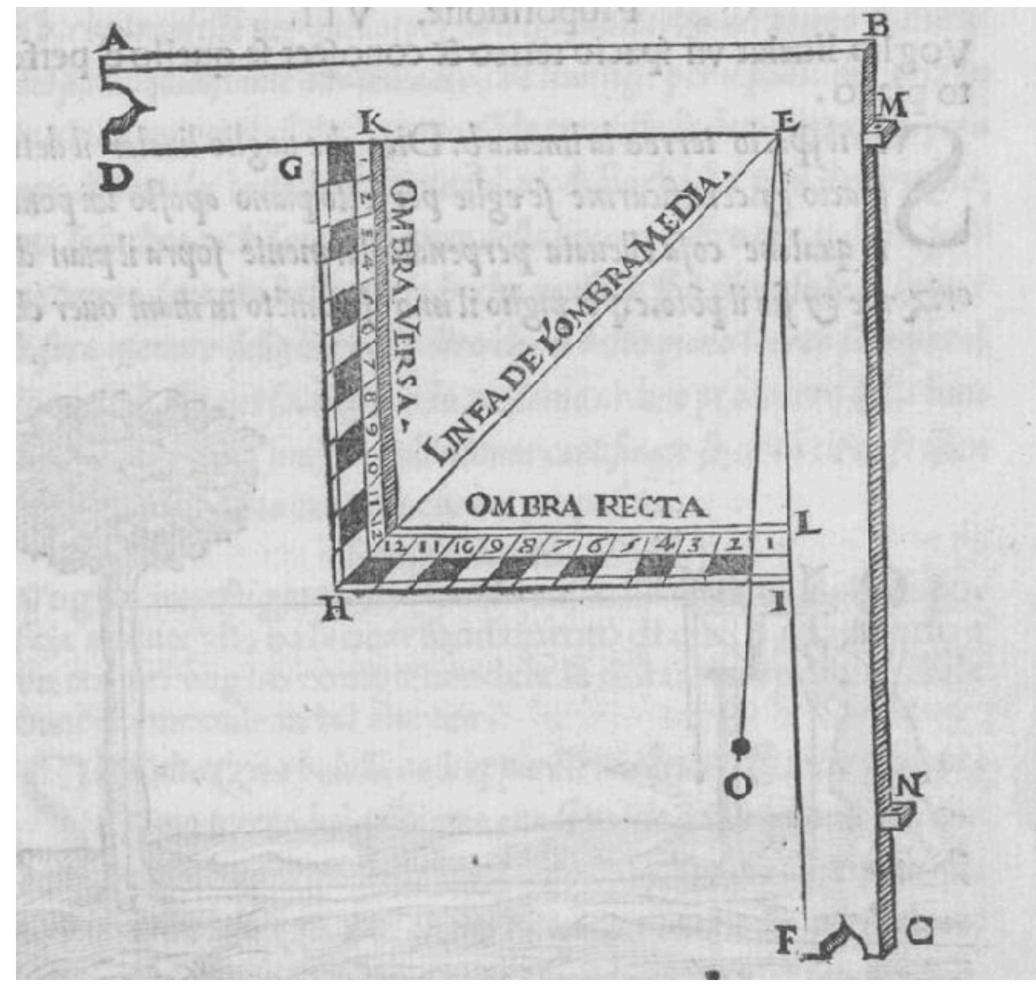

**Leon Battista Alberti “Ex Ludis rerum mathematicarum” 16th century**

These **applications** of mathematical tools were different from those devised by Tartaglia.

Measurement **records** properties of the World, Tartaglia’s applications to ballistics aimed at **influencing the world**, not merely observing it.

The first main contribution of Tartaglia consisted in **perfecting the quadrant** and developing a **systematic methodology** for its use in ballistics.

Tartaglia did not invent the quadrant. A quadrant can be seen as an enhanced equilibra, and equilibra had been in use for a long time.

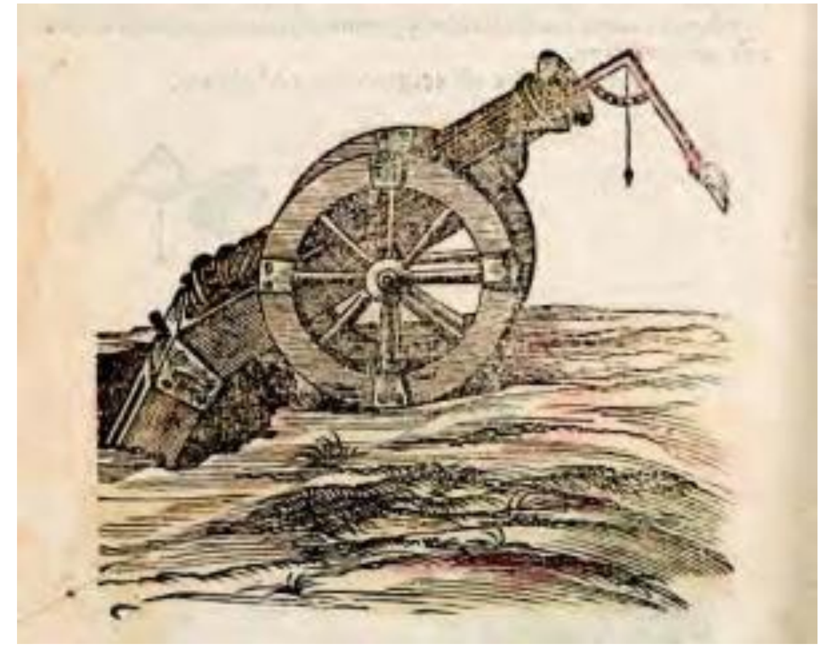


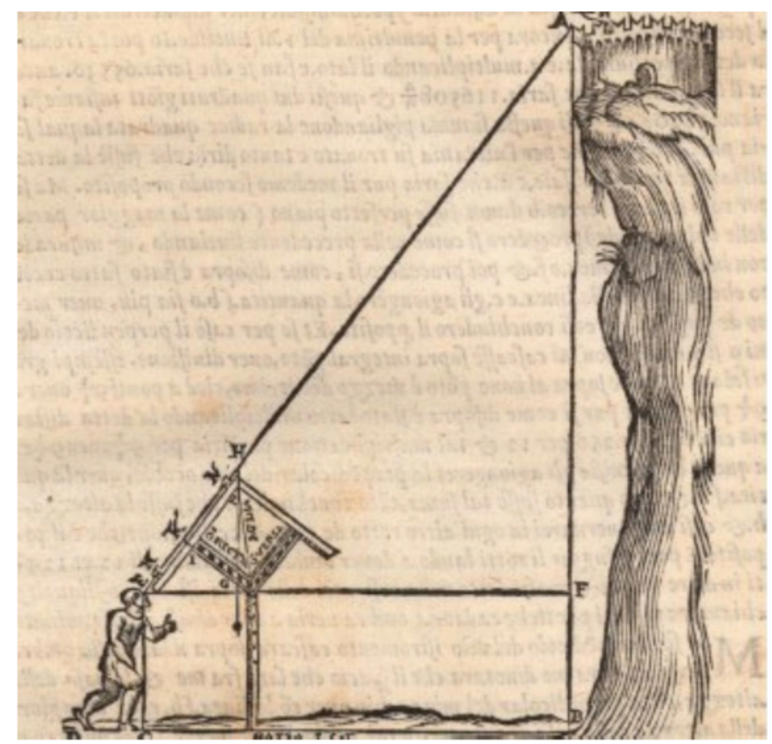

**Niccolò Tartaglia, "La Nova Scientia" (1537)**

In Tartaglia’s work the quadrant was described in two versions.

**One developed respectively for aiming cannon** with a longer leg to be inserted into the gun mouth and a quarter-circle divided into 12 points with plumb line —

**One developed to measure the height and distance of a target**. In the latter case, the quadrant has legs of the same length and a shadow square with plumb line.





**Niccolò Tartaglia, “La Nova Scientia” (1537)**

Prior to the era of ballistics the quadrant was chiefly used as an **instrument of recording**.

Before each shot the angle of elevation would be **measured** and **noted**.

If the shot was successful the annotation would be used to **realign** the position of the piece of artillery, which would have been lost through recoil.

[this procedure is discussed in Williams et al., 2010, p. 48-50].

Tartaglia’s work is not about mere recording and repositioning.

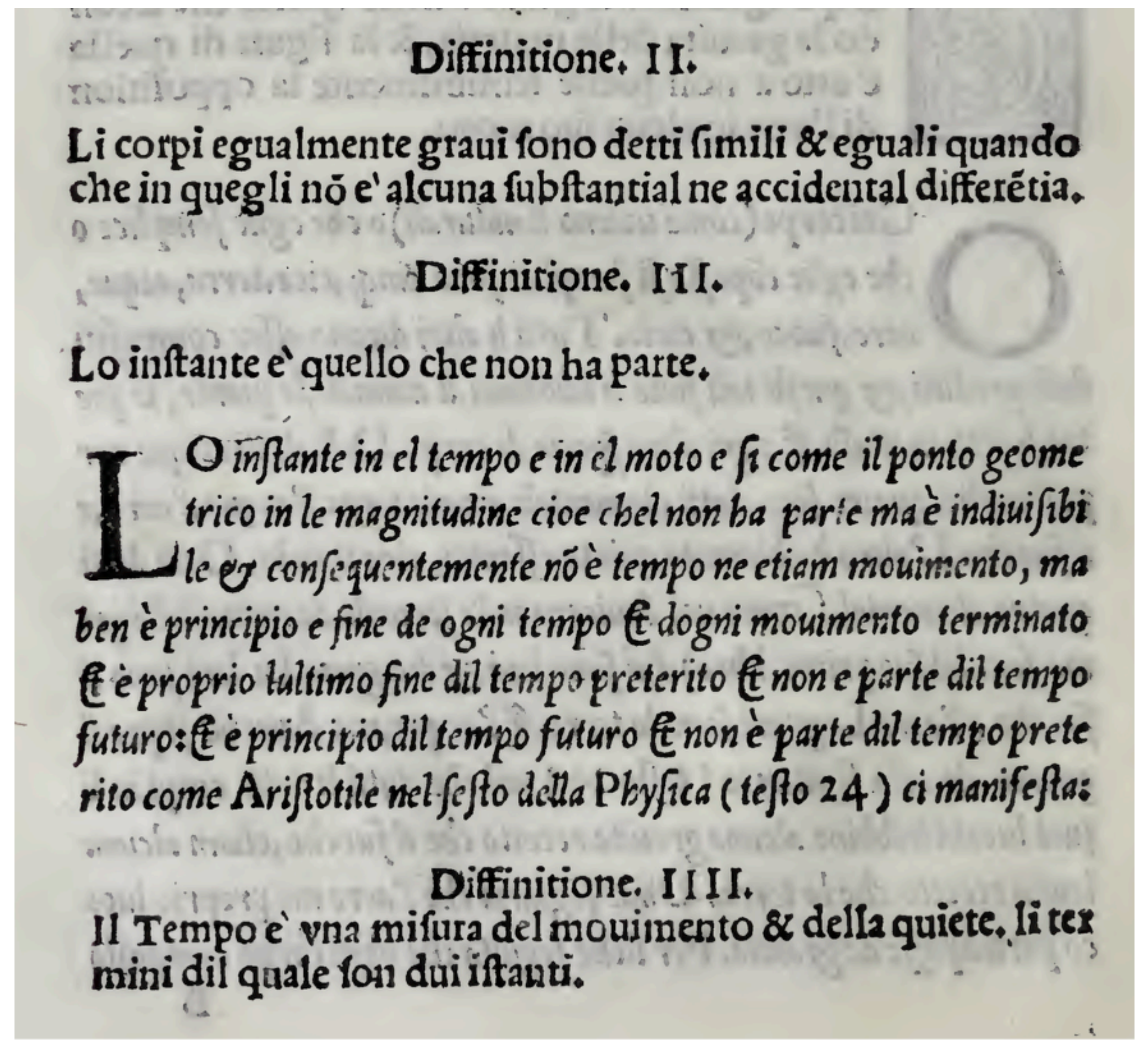
He illustrates techniques for **estimating distances** of target and **aiming cannons** accordingly.

These techniques are **based on Euclidean geometry** and are illustrated in form of **propositions** similar to the ones found in Euclid’s Elements.

Results presented are **supported by geometrical** (mathematical) **reasoning**.

Tartaglia’s explicit aim was to create a science that was strictly mathematical and of an **axiomatic-deductive nature.**

He begins with **definitions**, **postulates** and **axioms**.



Diffinitione. II.

Li corpi egualmente graui ſono detti ſimili & eguali quando che in quegli nõ e' alcuna ſubſtantial ne accidental differẽtia.

Diffinitione. III.

Lo inſtante e' quello che non ha parte.

LO inſtante in el tempo e in el moto e ſi come il ponto geometrico in le magnitudine cioe chel non ha parte ma è indiuiſibile & conſequentemente nõ è tempo ne etiam mouimento, ma ben è principio e fine de ogni tempo & dogni mouimento terminato & è proprio lultimo fine dil tempo preterito & non e parte dil tempo futuro: & è principio dil tempo futuro & non è parte dil tempo preterito come Ariſtotile nel ſeſto della Phyſica ( teſto 24 ) ci manifeſta:

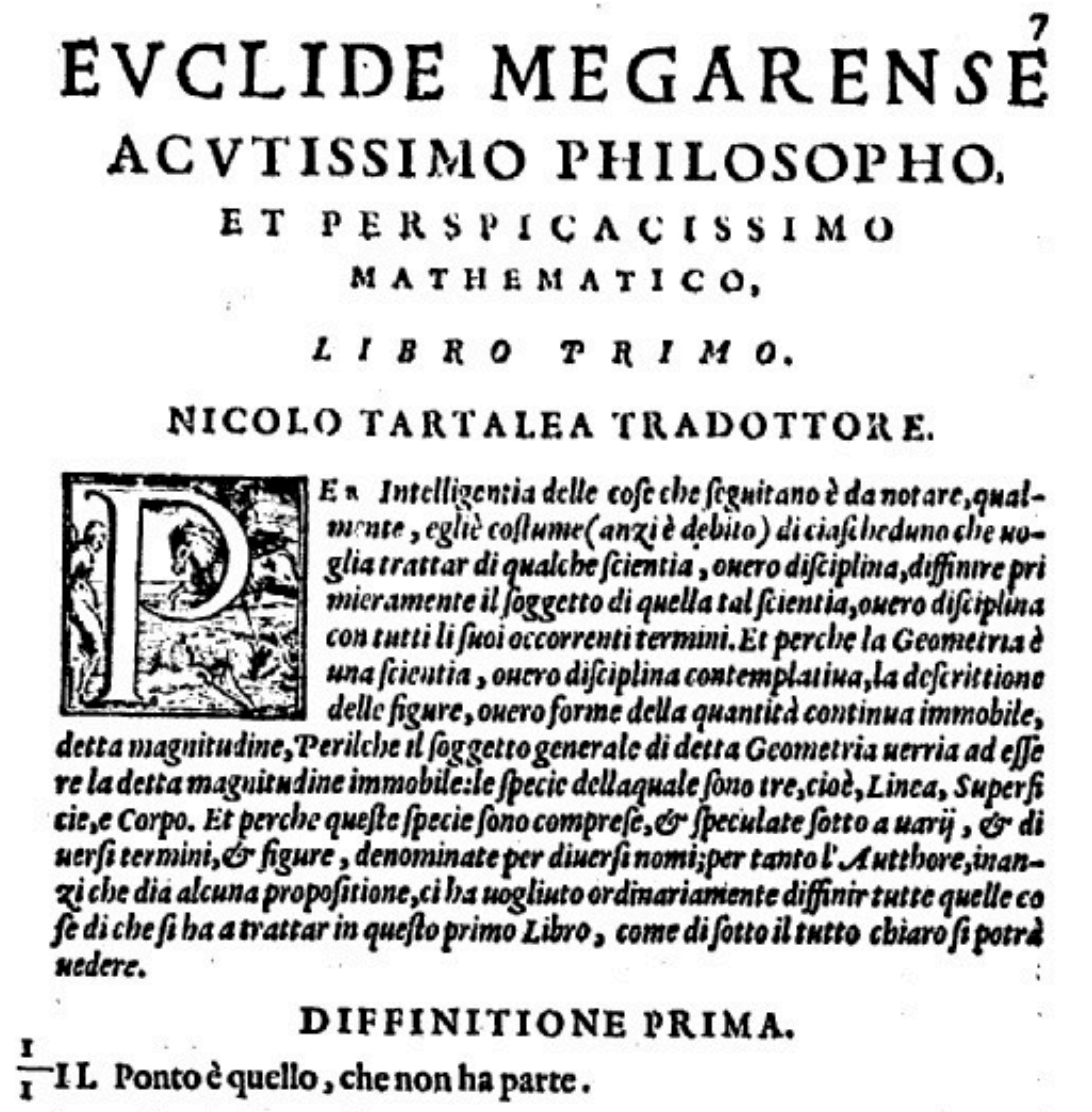
Diffinitione. IIII.

Il Tempo e' vna miſura del mouimento & della quiete, li termini dil quale ſon dui iſtanti.

**Niccolò Tartaglia, “La Nova Scientia” (1537)**

The **influence** of Euclid is apparent.

In fact, Tartaglia also wrote the first **translation of Euclid** in vernacular Italian.



7

EVCLIDE MEGARENSE

ACVTISSIMO PHILOSOPHO,

ET PERSPICACISSIMO MATHEMATICO,

*LIBRO PRIMO.*

NICOLO TARTALEA TRADOTTORE.

*Per Intelligentia delle coſe che ſeguitano è da notare, qualmente, egliè coſtume (anzi è debito) di ciaſcheduno che uoglia trattar di qualche ſcientia, ouero diſciplina, diffinire primieramente il ſoggetto di quella tal ſcientia, ouero diſciplina con tutti li ſuoi occorrenti termini. Et perche la Geometria è una ſcientia, ouero diſciplina contemplatiua, la deſcrittione delle figure, ouero forme della quantità continua immobile, detta magnitudine, Perilche il ſoggetto generale di detta Geometria uerria ad eſſere la detta magnitudine immobile: le ſpecie dellaquale ſono tre, cioè, Linea, Superficie, e Corpo. Et perche queſte ſpecie ſono compreſe, & ſpeculate ſotto a uarij, & diuerſi termini, & figure, denominate per diuerſi nomi; per tanto l'Autthore, inanzi che dia alcuna propoſitione, ci ha uogliuto ordinariamente diffinir tutte quelle coſe di che ſi ha a trattar in queſto primo Libro, come di ſotto il tutto chiaro ſi potrà uedere.*

DIFFINITIONE PRIMA.

$\frac{1}{1}$ IL Ponto è quello, che non ha parte.

**Niccolò Tartaglia, "Euclid's Elements," (1543)**

Finally **propositions** and **corollaries** emerge through a process of **deductive reasoning**.

There is hardly any difference from this structure and that of a modern OR research article.

Propositione. II.

*Tutti li corpi egualmente graui simile, & eguali dal principio delli lor mouimenti naturali, se partiranno de egual uelocita, ma giongendo al fine di tali lor mouimenti, quello che hauera passato per piu longo spacio andara piu ueloce.*

*Sel fusse le quatro diuerse altezze.a.b.et.c.d.poste a due a due in retta linea come disotto appare, et che la altezza.a.fusse tāto lontana dalla al*

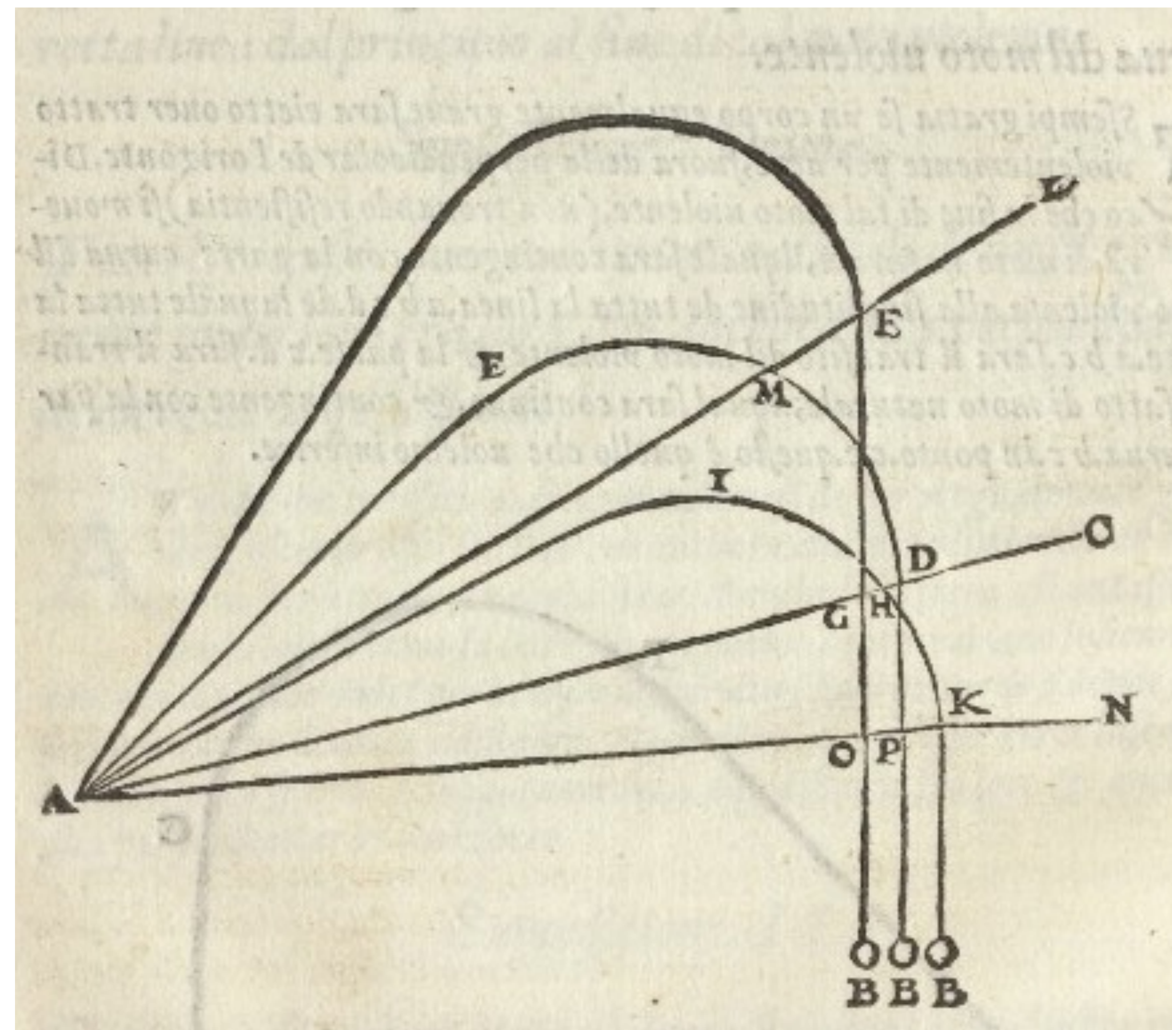


**Niccolò Tartaglia, “La Nova Scientia” (1537)**

Tartaglia had to operate within the **conceptual framework** of **Aristotelian physics**, as no other viable framework existed.

In line with Aristotelian physics, he described the trajectory (“transit”) of projectiles as a sequence of a **violent motion** followed by a **natural motion**, which were connected by a circular phase.

Tartaglia had **limited formal tools** to model trajectories mathematically, so he did what every OR person would do: he forced the trajectory to take an **approximate form** that he could analyse with the formal tools he had.

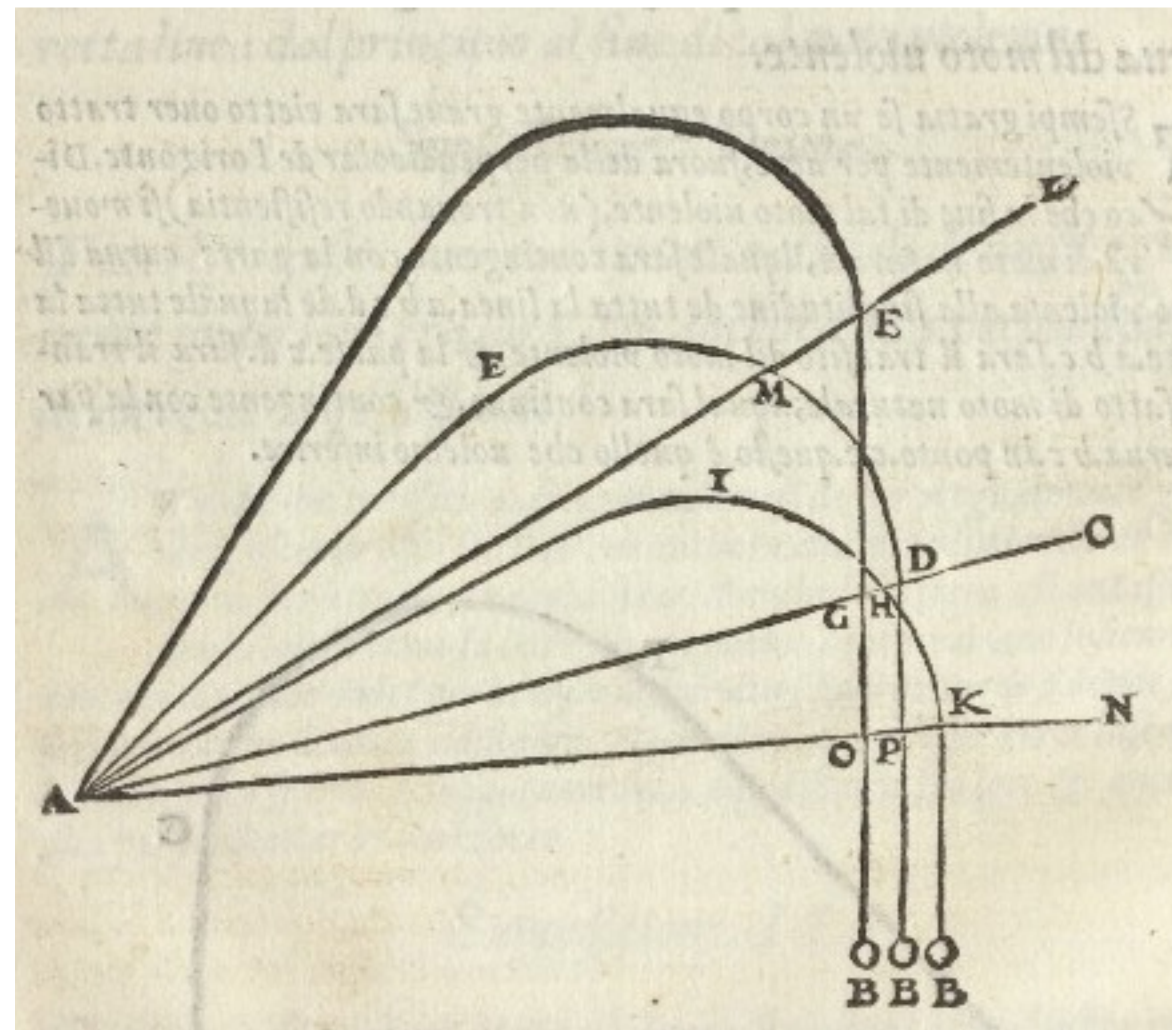



**Niccolò Tartaglia, “La Nova Scientia” (1537)**

Tartaglia knew that the real trajectory was not made up of two straight lines joined up by an arc of circumference.

He therefore states:

> "Nevertheless, that part [of the transit] that is not perceived as being curved is **assumed** to be straight, and that part that is evidently curved is **assumed** to be part of the circumference of a circle, **as this [assumption] does not influence the argument.**"




**Niccolò Tartaglia, "La Nova Scientia" (1537)**

On the one hand, Tartaglia strived to achieve the greatest possible **abstraction** of the practical problems he was facing and from the experience of the bombardiers, and thus to construct an **exact science** based on the Euclidean model.

On the other, Tartaglia was aware of the **applied nature** of the Nova Scientia he was introducing.

Tartaglia's arguments "cannot be explained on a purely geometrical basis, they require **observation and experience** in order to be considered valid." [Valleriani, 2013]

"Some previous knowledge of statistics is assumed and preferably some understanding of the role of statistical methods in applications; the latter understanding is important because many of the considerations involved are essentially conceptual rather than mathematical and relevant experience is necessary to appreciate what is involved."

*–D. Cox, ``Principles of Statistical Inference''*

Finally, it is worth spending some words on Tartaglia’s use of the quadrant as an **epistemic instrument**.

“The annotation regarding the angle of elevation of the piece of artillery is a first step in a process of abstraction, and therefore in theoretical reflection on the bombardier’s own actions.”

– *Valleriani (2013)*

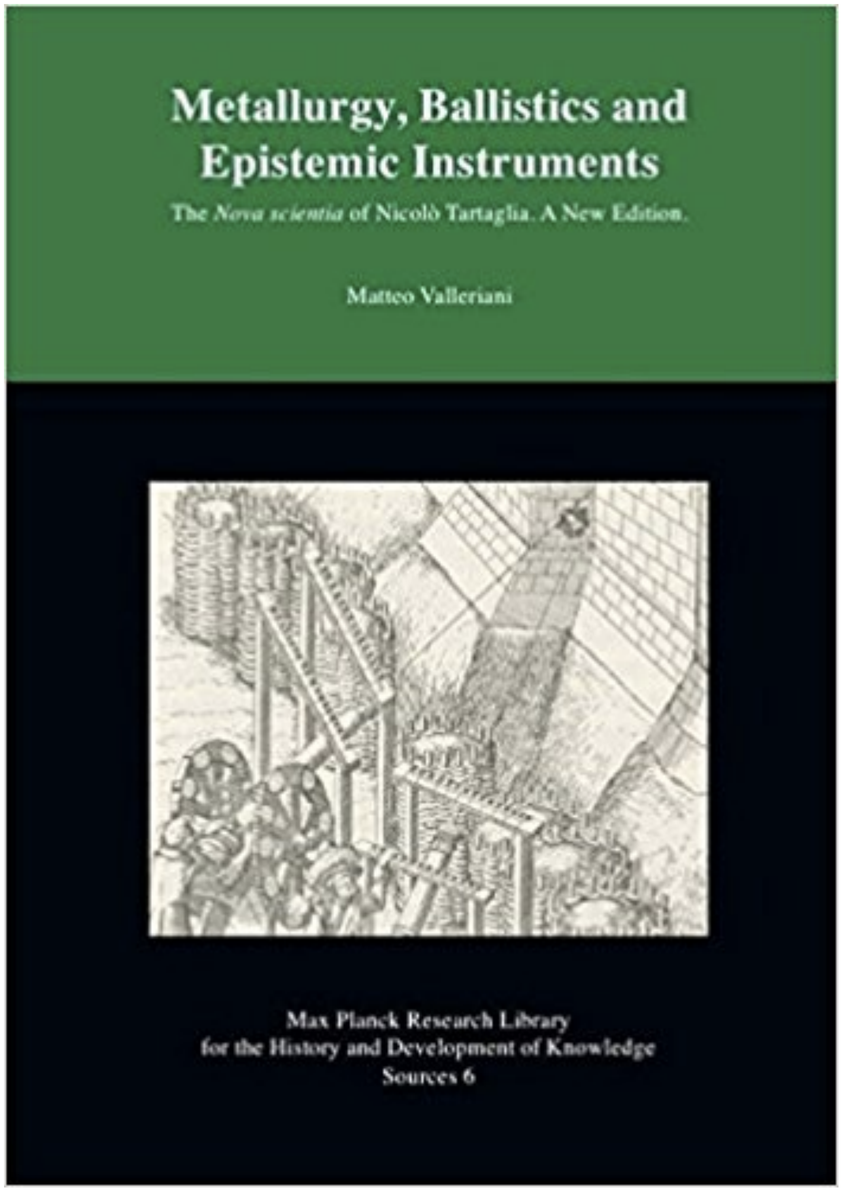
The **quadrant** links **theory** and **practice** and enables the transition from bombardier's experience to **ballistics** as a new theoretical subject.

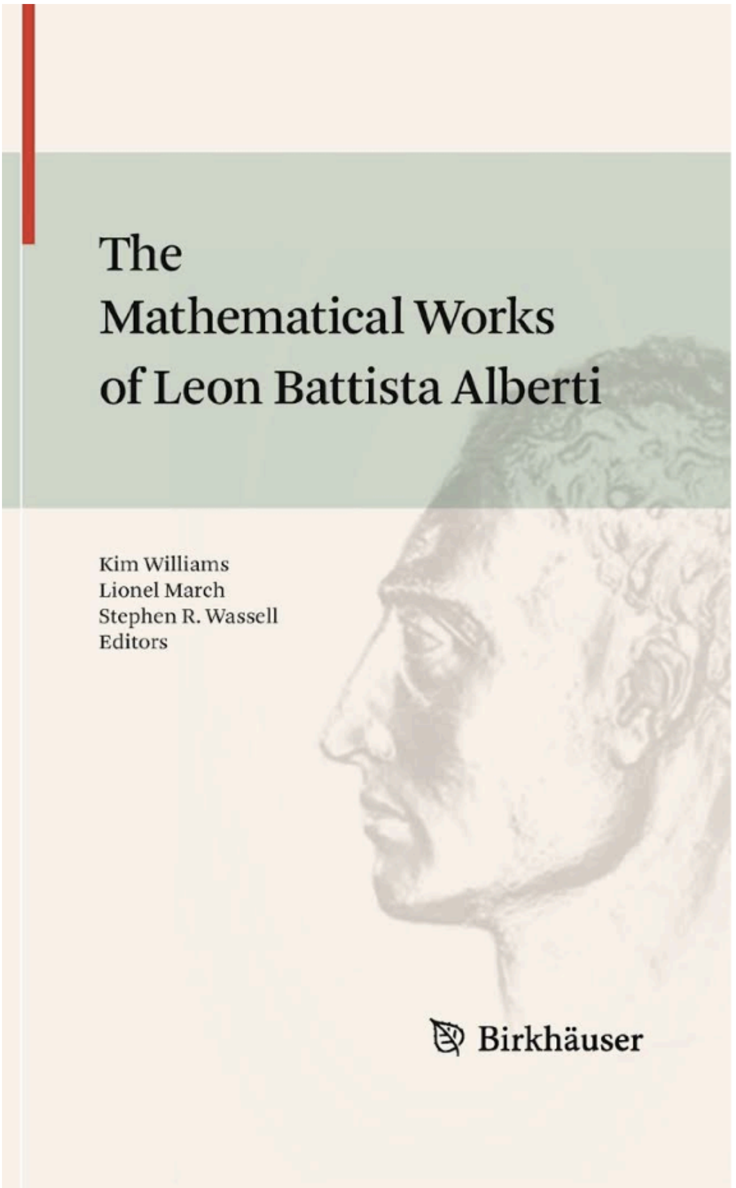
This process of **abstraction** makes OR

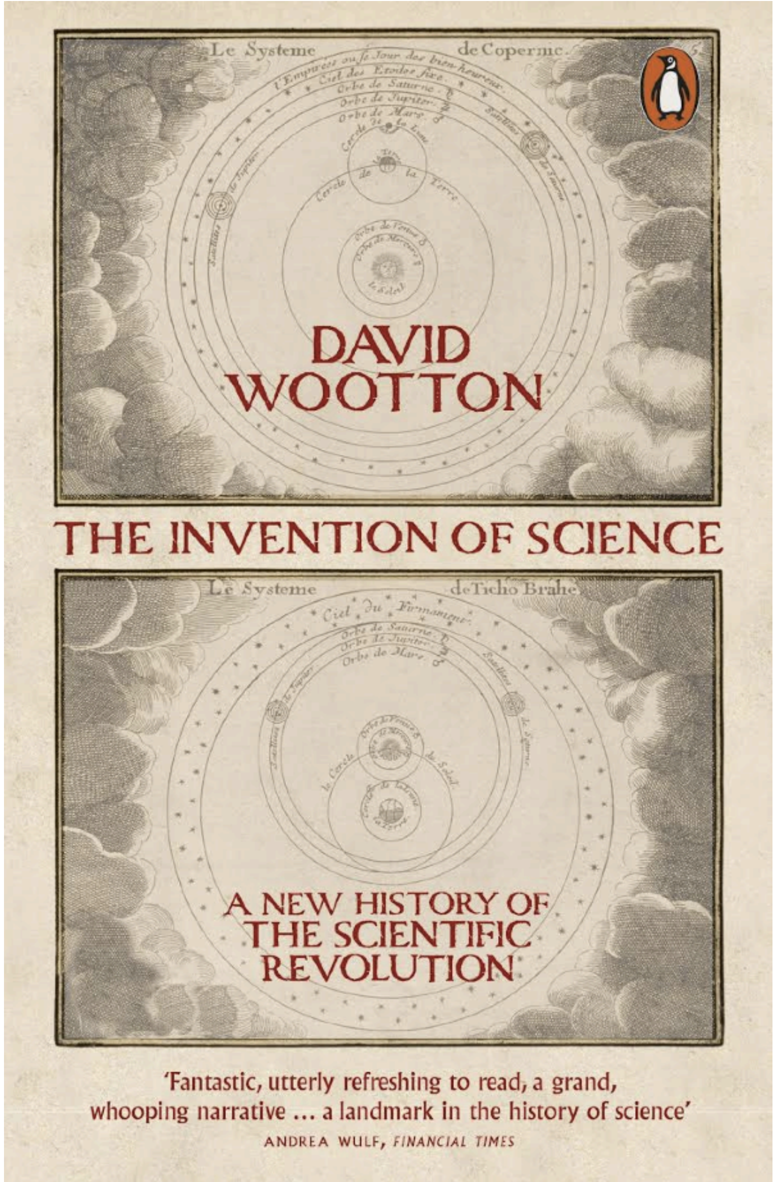
> "*the application of the methods of science to complex decision or optimisation problems arising in practical settings*"

**possible**.

# Essential bibliography



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