La Nova Scientia: Rewriting the History of Operational Research

Roberto Rossi

*(Submitted on 8 Mar 2017 (v1), last revised 6 Aug 2017 (this version, v5))*

The accepted lore is that Operational Research traces its roots back to the First and Second World Wars, when scientific research was used to improve military operations. In this essay we provide a different perspective on the origins of Operational Research by arguing that these are deeply intertwined with the impressive technological advances in Western Europe between the fifteenth and the sixteenth centuries.

Subjects: History and Overview (math.HO)

Cite as: arXiv:1703.03005 [math.HO]
(or arXiv:1703.03005v5 [math.HO] for this version)
“Operational Research (OR) is the application of the methods of science to complex problems arising in the direction and management of large systems of men, machines, materials, and money in industry, business, and defence.”

The accepted lore is that OR traces its roots back to the First and Second World Wars, when scientific research was used to improve military operations.


Optimal **positioning of radar bases** along the UK coastline during the Battle of Britain.

Military **resource allocation problems** in the context of the trade-off between aerial antisubmarine-warfare and strategic bombing operations in the Continent.

**Cryptanalytic techniques** such as Banburismus and Scritchmus developed at Bletchley Park.
“Whilst the discipline and practice of OR originated in the later 1930s and came to fruition during the Second World War, the history of warfare is punctuated by attempts to apply some elements of quantitative analysis to understanding the causes of victory and defeat…”

“...the first overt attempt to apply algebra and numbers to the battlefield occurred in later sixteenth century.”

“...as soon as the first rank has fired together, [...] they will march to the back. The second rank, either marching forward or standing still, [will next] fire together [and] then march to the back. After that, the third and following ranks will do the same. Thus before the last ranks have fired, the first will have reloaded.”

–Letter from Louis to Maurice (Counts of Nassau, 1585 - 1625)
However, Kirby defines Maurice as an “unconscious practitioners” of OR.

A natural question then arises...

Were military practitioners of the sixteenth century simply following common sense, or did some of them follow a principled, scientific approach akin to the one in use today?

What would happen if we read their research works today?
The fifteenth and sixteenth centuries saw impressive technological advances.
“Painting, cartography and ballistic do not strike us as cutting-edge sciences, but once they were.”

Painting

Il “terremoto di Efeso,” abside chiesa di San’t Agostino, Rimini, Italy (c.a 1350)
La città ideale, autore sconosciuto (1480-90)
As Brunelleschi (1377–1446) made no written record of his perspective findings, it remained for L. B. Alberti to be the first to put the theory into writing, in his treatise on painting, "Della Pittura" (1435).

Alberti described how an artist could get a correct view of a scene by observing it through a thin veil, or velo.
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Cartography

The Carta Pisana is the oldest surviving “portolano,” i.e. nautical chart.

It is not simply a map but a document showing accurate navigational directions.

Carta Pisana portolan chart (1275–1300)
The Carta Pisana is the oldest surviving “portolano,” i.e. nautical chart.

It is not simply a map but a document showing accurate navigational directions.

Best used in conjunction with a compass!

Carta Pisana portolan chart (1275–1300)
Mercator's projection turns sailing courses of a constant bearing into straight lines on the map thus greatly facilitating navigation.
Ballistics

Valleriani (2013) investigated how and why a new theoretical ballistic had emerged in the sixteenth century.

Equilibrium between attack and defence strategies that had endured for centuries has been destroyed by introduction of gunpowder/heavy artillery.

The emergence of new theoretical knowledge should be intended as a consequence of an advanced and challenging technological context.
Both Kirby (2003) and Valleriani (2013) date early attempts to apply algebra and numbers to the battlefield in the late sixteenth century.

But how did it all started and what are the essential ingredients (if any) to the OR recipe?
The decline and collapse of the Byzantine empire heightened contact between its scholars and those of the west and brought an influx of Neoplatonic scholars.

Constantine XI Dragases Palaiologos, (8 February 1405 – 29 May 1453) was the last reigning Byzantine Emperor
Georgius Gemistus (c. 1355–1452/1454) **reintroduced** Plato’s thoughts to Western Europe during the 1438–1439 Council of Florence.

He founded a new Platonic Academy that focused on the **translation** of Plato into Latin.
As a consequence of the Renaissance revival of Greek mathematics and of the rational tradition of Greek science, we find a proliferation of studies and translations of Greek works, including Euclid.

Euclid's Elements, in Latin, published by Erhard Ratdolt on May 25, 1482, in Venice
Euclid’s *Elements* was translated into Arabic in the ninth century.

Muslim mathematicians then combined geometry with Hindu arithmetic and algebra and developed new advances.

In the twelfth century the work was **translated** into Latin, making it more accessible to European scholars.

*Euclid's Elements, in Latin, published by Erhard Ratdolt on May 25, 1482, in Venice*
Neoplatonism influence was strong at the time of Leon Battista Alberti (1404–1472).

Mathematics, as a discipline, tended to be framed in a Neoplatonic context.


L. B. Alberti, Tempio Malatestiano (1450), Rimini, Italy
In his treatise “On Painting” Leon Battista Alberti wrote:

“Mathematicians measure the shapes and forms of things in the mind alone and divorced entirely from matter.”

Reinassance Platonists had been interested in number mysticism rather than real mathematics (Yates, 2002).
A key ingredient of the OR recipe was missing:

the connection between
theoretical models and practical applications
The development of this connection is what motivated the studies discussed in Tartaglia’s “La Nova Scientia” (1537)
In 1532, while he was living in Verona, a friend who was a bombardier asked Tartaglia:

**At which angle the barrel of a cannon should be elevated to achieve the longest possible shot?**

Niccolò Tartaglia, “La Nova Scientia” (1537)
Tartaglia did not have expertise in specialised areas connected to military activity.

However, having made some calculations, he was able to establish on geometric and algebraic grounds that the maximum range would be achieved if the barrel of a cannon were raised at an angle of 45 degrees above the line of horizon.

Niccolò Tartaglia, “La Nova Scientia” (1537)
By answering this question, Tartaglia **consciously engaged** in an enquiry that today we would have no problem in labelling as OR.
In Tartaglia’s time there had been a proliferation of mathematical treatises on topics such as algebra and combinatorics.

Both Ramon Lull and Gerolamo Cardano wrote an “Ars Magna.”

Cardano's Ars Magna ("The Great Art") published in 1545
What made Tartaglia’s work new (hence the title “Nova Scientia”) was the application of abstract mathematical models to achieve practical outcomes.

This was a revolutionary step that paved the way and inspired - as it is clear from the title of his treatise “Discorsi intorno a due nuove scienze” - Galileo’s works.
In the frontispiece of his work Tartaglia provides a motto stating that that the mathematical disciplines are seen as the only method to understand “the reasons of things” and that “is open to every one;”

This sets a precedent to Galileo’s Book of Nature and represents a clear cut from the Hermetic and Neoplatonic traditions, in which knowledge is esoteric.
However, the connection between theoretical models and applications requires a further ingredient not yet discussed...
However, the connection between theoretical models and applications requires a further ingredient not yet discussed:

measurement.
Influenced by Euclid’s work, Leon Battista Alberti in his work “Ex Ludis rerum mathematicarum” discussed applications of trigonometry to surveying (Williams et al., 2010, 2013).

The methods discussed relied on instruments called equilibra. Equilibra are a simple extension of the plumb line and could be used to measure angles in everyday activities.
These applications of mathematical tools were different from those devised by Tartaglia.

Measurement records properties of the World, Tartaglia’s applications to ballistics aimed at influencing the world, not merely observing it.
The first main contribution of Tartaglia consisted in perfecting the quadrant and developing a systematic methodology for its use in ballistics.

Tartaglia did not invent the quadrant. A quadrant can be seen as an enhanced equilibra, and equilibra had been in use for a long time.
In Tartaglia’s work the quadrant was described in two versions.

One developed respectively for aiming cannon with a longer leg to be inserted into the gun mouth and a quarter-circle divided into 12 points with plumb line —

One developed to measure the height and distance of a target. In the latter case, the quadrant has legs of the same length and a shadow square with plumb line.

Niccolò Tartaglia, “La Nova Scientia” (1537)
Prior to the era of ballistics the quadrant was chiefly used as an **instrument of recording**.

Before each shot the angle of elevation would be **measured** and **noted**.

If the shot was successful the annotation would be used to **realign** the position of the piece of artillery, which would have been lost through recoil.

[this procedure is discussed in Williams et al., 2010, p. 48-50].
Tartaglia’s work is not about mere recording and repositioning.

He illustrates techniques for estimating distances of target and aiming cannons accordingly.

These techniques are based on Euclidean geometry and are illustrated in form of propositions similar to the ones found in Euclid’s Elements.

Results presented are supported by geometrical (mathematical) reasoning.
Tartaglia’s explicit aim was to create a science that was strictly mathematical and of an axiomatic-deductive nature.

He begins with definitions, postulates and axioms.

Niccolò Tartaglia, “La Nova Scientia” (1537)
The influence of Euclid is apparent.

In fact, Tartaglia also wrote the first translation of Euclid in vernacular Italian.

Niccolò Tartaglia, “Euclid’s Elements,” (1543)
Finally **propositions** and **corollaries** emerge through a process of **deductive reasoning**.

There is hardly any difference from this structure and that of a modern OR research article.

Niccolò Tartaglia, “La Nova Scientia” (1537)
Tartaglia had to operate within the conceptual framework of Aristotelian physics, as no other viable framework existed.
In line with Aristotelian physics, he described the trajectory (“transit”) of projectiles as a sequence of a **violent motion** followed by a **natural motion**, which were connected by a circular phase.

Tartaglia had **limited formal tools** to model trajectories mathematically, so he did what every OR person would do: he forced the trajectory to take an **approximate form** that he could analyse with the formal tools he had.

Niccolò Tartaglia, “La Nova Scientia” (1537)
Tartaglia knew that the real trajectory was not made up of two straight lines joined up by an arc of circumference.

He therefore states:

“Nevertheless, that part [of the transit] that is not perceived as being curved is assumed to be straight, and that part that is evidently curved is assumed to be part of the circumference of a circle, as this [assumption] does not influence the argument.”

Niccolò Tartaglia, “La Nova Scientia” (1537)
On the one hand, Tartaglia strived to achieve the greatest possible abstraction of the practical problems he was facing and from the experience of the bombardiers, and thus to construct an exact science based on the Euclidean model.

On the other, Tartaglia was aware of the applied nature of the Nova Scientia he was introducing.

Tartaglia’s arguments “cannot be explained on a purely geometrical basis, they require observation and experience in order to be considered valid.” [Valleriani, 2013]
“Some previous knowledge of statistics is assumed and preferably some understanding of the role of statistical methods in applications; the latter understanding is important because many of the considerations involved are essentially conceptual rather than mathematical and relevant experience is necessary to appreciate what is involved.”

–D. Cox, “Principles of Statistical Inference”
Finally, it is worth spending some words on Tartaglia’s use of the quadrant as an epistemic instrument.
“The annotation regarding the angle of elevation of the piece of artillery is a first step in a process of abstraction, and therefore in theoretical reflection on the bombardier’s own actions.”

– Valleriani (2013)
The quadrant links theory and practice and enables the transition from bombardier’s experience to ballistics as a new theoretical subject.
This process of abstraction makes OR

“the application of the methods of science to complex decision or optimisation problems arising in practical settings”

possible.
Essential bibliography


