A simple heuristic for perishable inventory control under nonstationary stochastic demand

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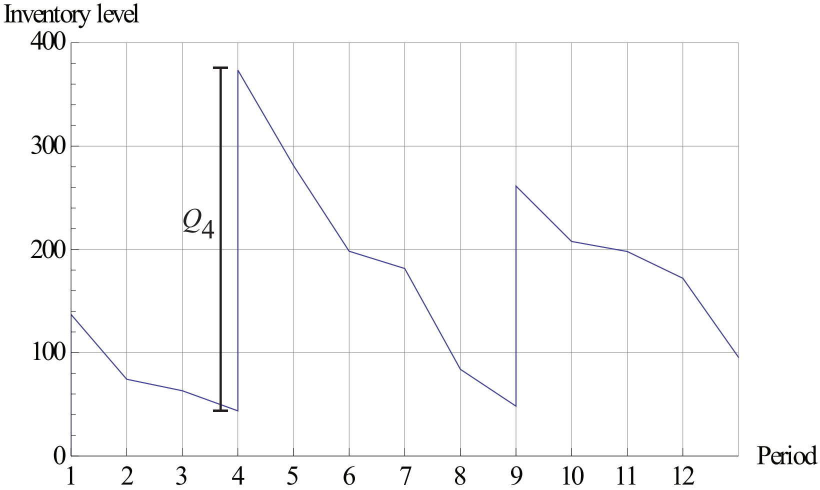
Around one third of the food that is produced worldwide for human consumption is lost or wasted every year (1.3 billion tons per year).

Food and Agriculture Organisation
of the United Nations (FAO)

http://www.fao.org/save-food/resources/keyfindings/en/

Stochastic lot sizing



Lot sizing

Dynamic deterministic lot sizing

Exact	**Heuristic**
Wagner and Whitin [1958]	Silver and Meal [1973]

Nonstationary stochastic lot sizing

Exact	**Heuristic**
SDP	Silver [1978]
	Askin [1981]
	Bookbinder and Tan [1988]
	Bollapragada and Morton [1999]
	Tarim and Kingsman [2006]
	Rossi et al. [2015]

Importance of considering nonstationarity: Tunc et al. [2011]

Lot sizing for a perishable item

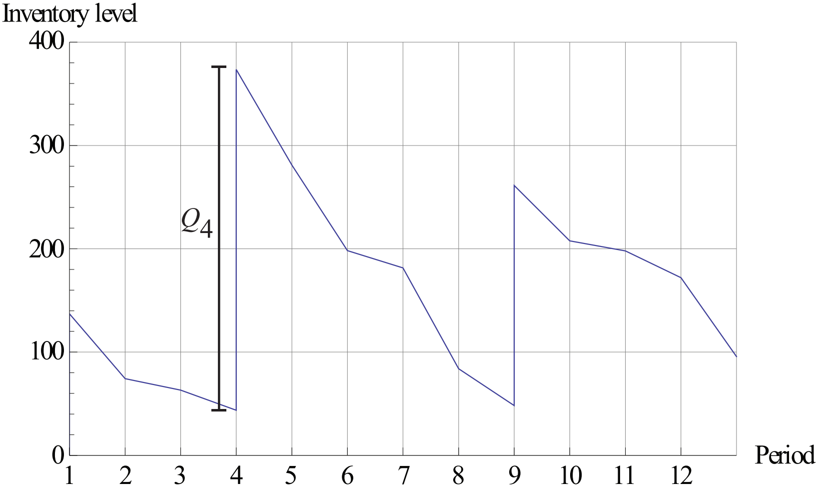
Literature surveys
Nahmias [1982]
Karaesmen et al. [2011]
Bakker et al. [2012]

Stochastic lot sizing for a perishable item
Exact
SDP
Heuristic
Minner and Transchel [2010]: no fixed ordering cost
Hendrix et al. [2012]: service level constraints
Pauls-Worm et al. [2014]: static-dynamic uncertainty policy
Alcoba et al. [2015]: static uncertainty policy

There is no other work operating under the assumptions of nonstationary demand, penalty cost and FIFO issuing policy.

Stochastic lot sizing for a perishable item

Key challenges



Contributions

We make the following contribution to the literature on perishable item non-stationary stochastic lot sizing.

- We introduce **exact analytical expressions** to compute the expected value of the inventory for different product ages when a product can age indefinitely; the expressions work under discrete as well as continuous demand distributions.
- We derive an **analytical approximation** for the case in which the product age is discrete and finite.
- By using the former results, we **extend Silver's heuristic** [Silver, 1978] to the case in which the product is perishable; in particular we introduce analytical and simulation-based variants of the approach.
- We conduct an **extensive numerical study** which demonstrates that our new heuristics lead to a cost that, on average, is 5% higher than the optimal cost.

Stochastic lot sizing

Stochastic lot sizing for perishable item

Indices

t	period index, $t = 1, \ldots, T$, with T the time horizon
a	age index, $a = 1, \ldots, A$, with A the fixed shelf life

Problem parameters

o	fixed ordering cost
v	unit procurement cost
h	unit inventory cost
p	unit penalty cost
w	unit disposal cost
i^a	initial inventory of age a at the beginning of the planning horizon

Random variables

D_t	random demand in period t with cumulative distribution function F_t

State variables

$\mathbf{I}_t$	system state (i.e. inventory vector) at end of period t
I_t^a	inventory of age a at end of period t, where $I_t^a \geq 0$ for $a = 2, \ldots, A$
W_t	waste at the end of period t (i.e. I_t^A)

Decision variables

Q_t	order quantity at the beginning of period t given inventory $I_{t-1}^1, \ldots, I_{t-1}^{A-1}$

Stochastic lot sizing for perishable item

Stochastic Dynamic Programming formulation

Decision stages. Planning horizon of T periods.

Actions. Order Q_t for period t given initial inventory $\mathbf{I}_{t-1}$.

State Transition Function.

Inventory of fresh products/backorders:
$$I_t^1 = Q_t - (D_t - \textstyle\sum_{a=1}^{A-1} I_{t-1}^a)^+, \quad t = 1, \ldots, T.$$

Inventory of age a:
$$I_t^a = \left(I_{t-1}^{a-1} - (D_t - \textstyle\sum_{j=a}^{A-1} I_{t-1}^j)^+\right)^+, \quad t = 1, \ldots, T; \quad a = 2, \ldots, A-1.$$

Waste:
$$W_t = (I_{t-1}^{A-1} - D_t)^+, \quad t = 1, \ldots, T.$$

Immediate Costs.
Given state $\mathbf{I}_{t-1}$ and the described transformation towards state $\mathbf{I}_t$ as a consequence of action Q_t, the immediate costs are given by

$$C(\mathbf{I}_{t-1}, Q_t) = g(Q_t) + \mathrm{E}\left(h \sum_{a=1}^{A-1} (I_t^a)^+ + p(-I_t^1)^+ + w\ W_t\right),$$

where E denotes the expectation taken with respect to random demand D_t and function g is defined as:

$$g(Q) = \begin{cases} o + vQ & \text{if } Q > 0 \\ 0 & \text{if } Q = 0. \end{cases}$$

Objective Function. The objective is to find an order policy that minimizes the expected total cost over the planning horizon

$$\min_{Q_1}\left(C(\mathbf{I}_0, Q_1) + \min_{Q_2}\left(C(\mathbf{I}_1, Q_2) + \cdots + \min_{Q_T} C(\mathbf{I}_{T-1}, Q_T)\right)\right).$$

Stochastic lot sizing for perishable item

Perishable stock dynamics

We assume that only I_t^1 can take a negative value, whereas older age inventory is bound to be nonnegative.

For convenience, we introduce $I_{t-1}^0 = Q_t$, so that we can denote the overall inventory available at the beginning of the period as

$$Y = \sum_{a=0}^{A-1} I_{t-1}^a;$$

we also exploit notation I_t^A to denote the waste W_t.

Our aim is to find analytical expressions for $\mathrm{E}(I_t^a)$, i.e. the expected value of I_t^a, for $a = 1, \ldots, A$.

Stochastic lot sizing for perishable item

Perishable stock dynamics

Lemma 1

Let D be a random variable defined over a continuous support and $Y = \sum_{a=0}^{A-1} I_{t-1}^{a}$,

$$\mathrm{E}(-I_t^1)^+ = \int_0^Y (Y - x) f(x)\,\mathrm{d}x - (Y - \mu).$$

Rossi et al. [2015]

Stochastic lot sizing for perishable item

Perishable stock dynamics

Lemma 2

Let D be a random variable defined over a discrete support $\mathbb{S} \subseteq \mathbb{N}$ with a positive probability mass function $f(x) = F(x) - F(x-1)$ for $x \in \mathbb{S}$ and zero elsewhere.

Let $Y_a = \sum_{j=a-1}^{A-1} I_{t-1}^j$, $Y_{a+1} = \sum_{j=a}^{A-1} I_{t-1}^j$, and $Y_{A+1} = 0$, then for $a = 1, \ldots, A$

$$\mathrm{E}(I_t^a)^+ = \sum_{x=0}^{Y_a - 1} F(x) - \sum_{x=0}^{Y_{a+1}-1} F(x).$$

Lemma 3

Let D be a random variable defined over a continuous support, $Y_a = \sum_{j=a-1}^{A-1} I_{t-1}^j$, $Y_{a+1} = \sum_{j=a}^{A-1} I_{t-1}^j$, and $Y_{A+1} = 0$, then for $i = 1, \ldots, A$

$$\mathrm{E}(I_t^a)^+ = \int_0^{Y_a} (Y_a - x) f(x)\,\mathrm{d}x - \int_0^{Y_{a+1}} (Y_{a+1} - x) f(x)\,\mathrm{d}x.$$

Stochastic lot sizing for perishable item

Perishable stock dynamics

Lemma 4

$$\mathrm{E}(I_t^a) = \mathrm{E}(I_t^a)^+ - \mathrm{E}(-I_t^a)^+.$$

Lemma 5

For $a = 2, \ldots, A$,

$$\mathrm{E}(I_t^a) = \mathrm{E}(I_t^a)^+.$$

Stochastic lot sizing for perishable item

Perishable stock dynamics: indefinite aging

Our aim is to compute $\mathrm{E}(I_t^a)$, i.e. the expected value of I_t^a, for $t = 1, \ldots, T$ and $a = t, \ldots, A + a - 1$.

We first analyse the case where *items can age indefinitely*.

This question is easy to handle if we reduce a t-period problem to a single-period problem subject to cumulative demand $D = D_1 + \ldots + D_t$.

Consider $\mathrm{E}(I_t^a)$ for this new single-period problem, as previously obtained, and note that the value obtained corresponds to $\mathrm{E}(I_t^{t+a-1})$ for the t-period problem.

Example

Consider a $T = 2$ periods planning horizon.

Demand D_t in each period t follows a Poisson distribution with rate $\lambda_t = 50$.

The initial inventory is $\mathbf{I}_{t-1} = (I^0_{t-1}, I^1_{t-1}, I^2_{t-1}) = (25, 50, 50)$. This means there are 25 fresh items that have just been produced; 50 items that were produced a period before and 50 items that were produced two periods before, which are still in stock at the beginning of the planning horizon.

To determine $\mathrm{E}(I^a_1)$, $a = 1, \ldots, A$ we apply the results just presented, in particular Lemma 2, which lead to $\mathrm{E}(I_1) = (25, 47.18, 2.81)$.

The value of $\mathrm{E}(I^a_2)$ for $a = 2, \ldots, A + 1$ can be derived considering the 2-period problem as a single-period problem subject to a Poisson demand with rate $\lambda = 100$ and we apply once more the results just presented and Lemma 2. This leads to the vector $\mathrm{E}(\mathbf{I}_2) = (0, 21.04, 3.98)$. Since we do not place an order in period 2, the expected number of fresh items at the end of period 2 is zero. Items of age 3 at the end of period 1 have aged.

Stochastic lot sizing for perishable item

Perishable stock dynamics: wastage

The case in which items of age A — i.e. items that were produced $A-1$ periods before — are discarded at the end of a period, complicates the exact analytical derivation of $\mathrm{E}(I_t^a)$ for $t=1,\ldots,T$ and $a=t,\ldots,A+t-1$.

The key intuition leading to our approximation is an inductive argument. Our base case is the determination of $\mathrm{E}(I_t^a)$, which can be carried out analytically for $t=1$ and $a=t,\ldots,A$ (with $I_t^A=W_t$) by using the results presented so far. In principle it is also possible to determine analytically the higher moments of the distribution of I_t^a, e.g. the variance $\mathrm{Var}(I_t^a)$ for $t=1$.

To approximate $\mathrm{E}(I_t^a)$, we first operate as if items can age indefinitely, i.e. they are never discarded, and we reduce the t-period problem to a single period problem with demand $D=D_1+\ldots+D_t+W_1+\ldots+W_{t-1}$, where distributions of $W_1,\ldots,W_{t-1}$ are derived at previous induction steps. Once this new demand distribution is obtained, values for $\mathrm{E}(I_j^a)$ for $j<t$ and $a=t,\ldots,A-1$ can be computed iteratively by reusing results developed so far.

Once more, one should note that the value $\mathrm{E}(I_t^a)$ obtained for this new single-period problem corresponds to $\mathrm{E}(I_t^{t+a-1})$ for the t-period problem; of course, $\mathrm{E}(I_t^a)=0$ for $a>A$.

Stochastic lot sizing for perishable item

Perishable stock dynamics: indefinite aging

Example

Consider a problem in which D_t follows a Poisson distribution with rate λ_t for $t = 1, \ldots, T$ and the shelf life is $A = 3$.

The approach is based on an approximation of the distribution of I_t^a by fitting an appropriate first moment (i.e. mean) to a Poisson distribution. This means that in period t, after having carried out the induction over periods $1, \ldots, t-1$, we will reduce the t-period problem to a single period problem subject to Poisson demand with expected value $\mathrm{E}(D_1) + \ldots + \mathrm{E}(D_t) + \mathrm{E}(W_1) + \ldots + \mathrm{E}(W_{t-1})$.

We refer to the same problem analysed in the previous example. However, now the shelf life is limited to $A = 3$. We first compute $\mathrm{E}(\mathbf{I}_1) = (25, 47.18, 2.81)$, for which the computation is identical to the one carried out under the previous example setting. By exploiting this information, and in particular $\mathrm{E}(I_1^3) = 2.81$, (expected waste in period 1) we reduce the two-period problem to a single period problem under Poisson demand with rate $\lambda = 50 + 50 + 2.81$ and exploit once more Lemma 2 to compute the approximation $(0, 19.47, 2.77)$. This vector is close to the actual vector $\mathrm{E}(\mathbf{I}_2) = (0, 20.219, 1.993)$ which can be found carrying out an exact convolution.

Stochastic lot sizing for perishable item

Analytical extension of Silver's heuristics

Data: the current period t; initial inventory $\mathbf{I}_{t-1}$ at the beginning of period t

Result: the optimal order quantity, $Q^* \geq 0$

$r \leftarrow t$;

$c^* \leftarrow \infty$;

do

  $Q \leftarrow \text{argmin}_{Q \geq 0}[C(\mathbf{I}_{t-1}, Q) + C(\mathbf{I}_t, 0) + \ldots + C(\mathbf{I}_{r-1}, 0)]$;

  $c \leftarrow (C(\mathbf{I}_{t-1}, Q) + C(\mathbf{I}_t, 0) + \ldots + C(\mathbf{I}_{r-1}, 0))/(r - t + 1)$;

  if $c \leq c^*$ **then**

    $c^* \leftarrow c$;

    $Q^* \leftarrow Q$;

  end

  $r \leftarrow r + 1$;

while $c \leq c^*$ *and* $r < t + A$;

Example

Consider an instance over a planning horizon of $T = 3$ periods, with shelf life $A = 3$. Demand in each period t is Poisson distributed with rate λ_t, where $\lambda \in \{4, 3, 3\}$. Fixed ordering cost o is set to 10, holding cost is set to 1, waste cost w is set to 2, penalty cost p is set to 5. Initial inventory is $\mathbf{I}_0 = (I_0^1, I_0^2) = (1, 1)$. We consider replenishment cycles (t, r) of increasing length:

- for $(t, r) = (1, 1)$ the optimal action is to not order ($Q = 0$) and use existing inventory to cover demand in period 1, the expected total cost per period of this policy is 10.67 — note that, if we decided to place an order, the optimal order quantity to cover demand in period 1 would be $Q = 3.96$ and the expected total cost per period would increase to 13.21;
- for $(t, r) = (1, 2)$ the optimal action is to place an order $Q = 6.04$, the expected total cost per period of this policy is 9.56, since this is less than 10.67 we proceed and consider the next possible replenishment cycle length;
- for $(t, r) = (1, 3)$ the optimal action is to order $Q = 7.99$, the expected total cost per period of this policy is 9.68; since this is greater than 9.56, we conclude that the optimal action in period 1 is to order $Q = 6.04$ and incur an expected total cost per period of 9.56. After observing actual demand in period 1, the procedure can be iterated to determine order quantities in following periods.

Stochastic lot sizing for perishable item

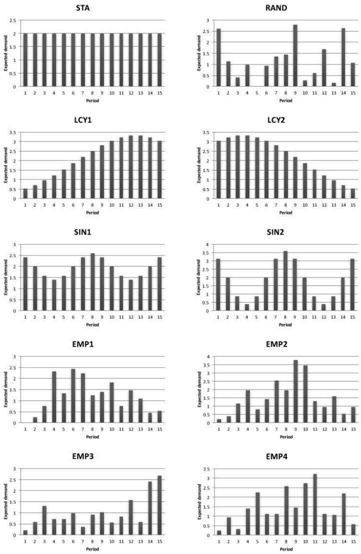
Simulation-optimisation heuristic

Data: the current period t; initial inventory $\mathbf{I}_{t-1}$ at the beginning of period t

Result: the optimal order quantity, $Q^* \geq 0$

$r \leftarrow t$;
$c^* \leftarrow \infty$;
do
  $Q \leftarrow \text{argmin}_{Q \geq 0}[\widehat{C}^N_{t,r}(\mathbf{I}_{t-1}, Q)]$;
  $c \leftarrow (\widehat{C}^N_{t,r}(\mathbf{I}_{t-1}, Q))/(r - t - 1)$;
  if $c \leq c^*$ **then**
    $c^* \leftarrow c$;
    $Q^* \leftarrow Q$;
  end
  $r \leftarrow r + 1$;
while $c \leq c^*$ *and* $r < t + A$;

Experimental design



Numerical results

	Analytical		Simulation-optimization		Observations
	MPE	0.95 CI	MPE	0.95 CI	
Ord cost level					
1	3.06	±1.77	2.91	± 1.66	18
2,5	5.24	±3.70	4.22	± 2.33	18
5	9.59	±6.36	7.14	± 3.84	18
Penalty cost					
2	2.37	±1.17	2.00	± 0.92	18
5	4.63	±3.04	4.63	± 2.78	18
10	10.8	±6.51	7.65	± 3.56	18
Waste					
1	1.74	± 0.59	2,13	± 0.75	18
5	3.68	± 2.03	3.44	± 1.60	18
10	12.4	± 6.45	8.71	± 3.99	18
Demand pattern					
EMP1	8.62	± 7.33	6.47	± 5.13	10
EMP2	3.53	±4.97	3.13	± 4.83	5
EMP3	8.78	±10.1	7.34	± 5.78	9
EMP4	23.8	±205	11.9	± 102	2
LCY1	2.42	±3.44	2.55	±3.27	5
LCY2	1.06	± 2.32	1.32	± 3.80	3
RAND	14.6	±12.1	12.9	± 21.1	2
SIN1	2.58	± 2.74	3.32	± 3.71	5
SIN2	3.20	± 2.97	2.91	± 2.31	5
STA	2.25	± 2.84	1.95	± 2.75	8
General	5.96	± 2.47	4.76	± 1.57	54
Time (aprox.)	5 secs	50 secs			

Conclusions

- We presented an inventory control model for perishable items with a fixed shelf-life;
- We discussed a number of analytical results concerning the dynamics of perishable stock in the inventory systems;
- We introduced two extensions of Silver's heuristic: an analytical approximation and a Monte Carlo simulation-optimisation approach;
- We demonstrated the effectiveness of these heuristics in an comprehensive numerical study;
- Our results show that the simulation-optimisation approach features a better cost performance that, on average, is 5% above the optimal cost, the respective figure for our analytical approximation is 6%;

Complete paper

The results here presented are discussed in

A. G. Alcoba, R. Rossi, B. Martin-Barragan, E. Hendrix, "A simple heuristic for perishable inventory control under nonstationary stochastic demand", International Journal of Production Research, Taylor & Francis, Vol. x(x):xxxx-xxxx, to appear, 2016

http://dx.doi.org/10.1080/00207543.2016.1193248

Conclusions

Questions