Multi-retailer stochastic inventory control with service level constraints and routing decisions

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Problem definition

- discrete finite time horizon
- multiple retailers
- single production facility
  - infinite production capacity
- one (or more) vehicle(s)
  - infinite loading capacity
- single product item
  - non-stationary stochastic demand
- back orders if stock out
- negative orders not allowed
- zero delivery lead times

- fixed transportation cost b/w any nodes
- fixed ordering cost
- unit ordering cost
- unit holding cost
- service level constraint
Problem definition (cont’d)

- **Objective**
  - To minimise the expected total costs incurred - these include:
    - vehicle routing costs
    - replenishment costs
    - inventory costs

- **Main Ingredients**
  - a vehicle routing component, i.e. essentially:
    - a Travelling Salesman Problem (if 1 vehicle)
    - a Clover Leaf Problem (if more than 1 vehicle)
  - a Stochastic Lot Sizing component - at each retailer

- **Inventory Control Policies**
    - Static Uncertainty (SU)
    - Dynamic Uncertainty
    - Static-Dynamic Uncertainty (RS)
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## Literature

### Deterministic

<table>
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<tr>
<th>Authors</th>
<th>Year</th>
<th>Demand</th>
<th>Routing</th>
<th>Lot-sizing</th>
<th>Technique</th>
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<td>Anily &amp; Federgruen</td>
<td>1990,1994</td>
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### Stochastic

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<td>Adelman</td>
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<td>Yu et al.</td>
<td>2012</td>
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Notation
Numerical study

- network notation
  - depot: node 1
  - retailers: nodes 2, ..., 6

- planning horizon
  - 12 periods

- demand
  - normally distributed
  - $\sigma/\mu = 0.2$

- initial stock
  - zero everywhere

- service level
  - $\alpha = 0.95$

- other parameters
  - $v = 0$
  - $h = 1$
Problem data - mean values of demand ($\mu$)

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Problem data - fixed transportation costs

Instance 1

Instance 2
RS: Instance 1

E[TRC]: 15857
Solution time: 20s (R)
758s
SU: Instance 1

E[TRC]: 16836
Solution time: 10.78s
RS: Instance 2

E[TRC]: 6362
Solution time: 2293s (R)
850s
SU: Instance 2

E[TRC]: 7885
Solution time: 62.59s
Ongoing Work

- Experimental campaign being completed
- So far - RS up to 30% better than SU
- Delivery lead times being incorporated