Cost-based Domain Filtering for Stochastic Constraint Programming

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1. **Introduction**
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   - Expectation-based relaxation for stochastic variables
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3. **Experiments**
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   - Stochastic sequencing with release times and deadlines
4. **Conclusions**
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   - Conclusions

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- **Stochastic Constraint Programming** is a framework for modeling combinatorial optimization problems that involve uncertainty (Walsh [7]).
- In this work, we show how to perform **cost-based filtering** for certain classes of **stochastic constraint programs**.

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Sample CSP

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A possible solution for the CSP is $x = 1$ and $y = 4$.

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Constraint Propagation

...is the **process** that repeatedly calls filtering algorithms until no new deduction can be made.

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during the search filtering algorithms are systematically applied when the domain of a variable is modified.

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  - they are associated with powerful **filtering strategies**
  - new **custom** global constraints can be defined

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These pieces of information are exploited both for **propagation** purposes and for **guiding the search**.

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- θ_h is a **threshold probability** associated with chance-constraint h

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- In general, the solution of an m**-stage stochastic CSP** is represented by means of a **policy tree** (Walsh [6])

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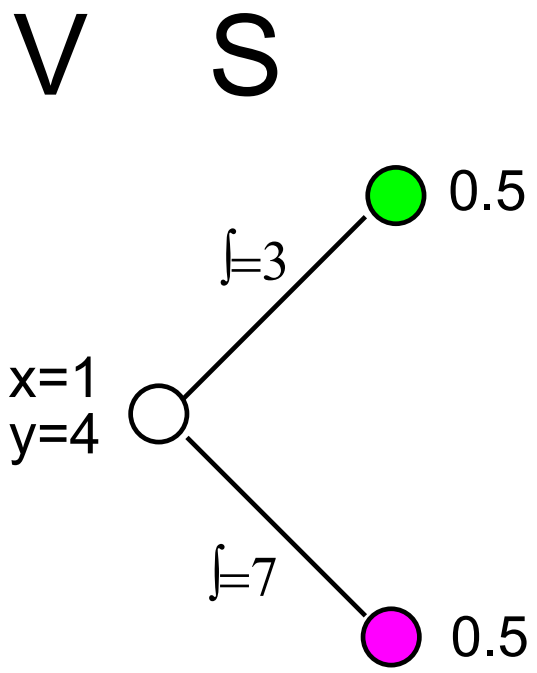
Sample Stochastic CSP

- $V = \{x, y\}$
- $S = \{\int\}$
- $D(x) = \{1, 3, 4, 5\}$ $D(y) = \{4, 5, 8\}$
- $P(\int) = \{3(50\%), 7(50\%)\}$
- $C = \{x + \int = y\}$
- $\theta = 50\%$

A possible solution for the CSP is $x = 1$ and $y = 4$



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- We may be interested in finding a feasible solution, i.e. a policy tree $s \in \mathcal{S}$, that **maximizes** (or minimizes) the value of a given objective function $f(\cdot)$ over the stochastic variables S (edges of the policy tree) and over a subset $\widehat{V} \subseteq V$ of the decision variables (nodes in the policy tree)

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- A **Stochastic COP** is then defined in general as $\max_{s \in \mathcal{S}} f(s)$

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Global Chance-Constraints

Perhaps the most interesting aspect of SCP is that the concept of **global constraint** can be also adopted in a stochastic environment, thus leading to

- Global Chance-Constraints (Rossi et al., 2008)

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- maximize the **expected profit** = total reward - penalty

Static Stochastic Knapsack

SSKP

We model the SSKP as a **one-stage** Stochastic COP, the recourse problem RP(SSKP) can be formulated as

Objective:

$$\max \left\{ \sum_{i=1}^{k} r_i X_i - c\mathbb{E}\left[\sum_{i=1}^{k} \mathcal{W}_i X_i - q \right]^+ \right\}$$

Decision variables:

(1) $X_i \in \{0, 1\}$ $\forall i \in 1, \ldots, k$

Stochastic variables:

$\mathcal{W}_i \rightarrow$ item i weight

Figure: RP(SSKP). Note that $[y]^+ = \max\{y, 0\}$.

Static Stochastic Knapsack

According to the approach described in Tarim et al. [6] we compile RP(SSKP) into a **deterministic equivalent** CP model using a **scenario-based** representation.

Objective:

$$\max \left\{ \sum_{i=0}^{k} r_i X_i - c \left[\sum_{j=1}^{n} Z_j \Pr\{j\} \right] \right\}$$

Constraints:

(1) $Z_j \geq \sum_{i=1}^{k} \mathcal{W}_i^j X_i - q \qquad \forall j \in 1, \ldots, n$

Decision variables:

(2) $X_i \in \{0, 1\} \qquad \forall i \in 1, \ldots, k$

(3) $Z_j \in [0, \sum_{i=1}^{k} \mathcal{W}_i^j] \qquad \forall j \in 1, \ldots, n$

Figure: DetEquiv(RP(SSKP)). $\Pr\{j\}$ is the probability of scenario $j \in \{1, \ldots, n\}$. Note that $\sum_{j=1}^{n} \Pr\{j\} = 1$.

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  $f(1) = \{10(0.5), 8(0.5)\}$, $f(2) = \{10(0.5), 12(0.5)\}$,
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- The other problem parameters are $c = 2$, $q = 30$.

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Static Stochastic Knapsack

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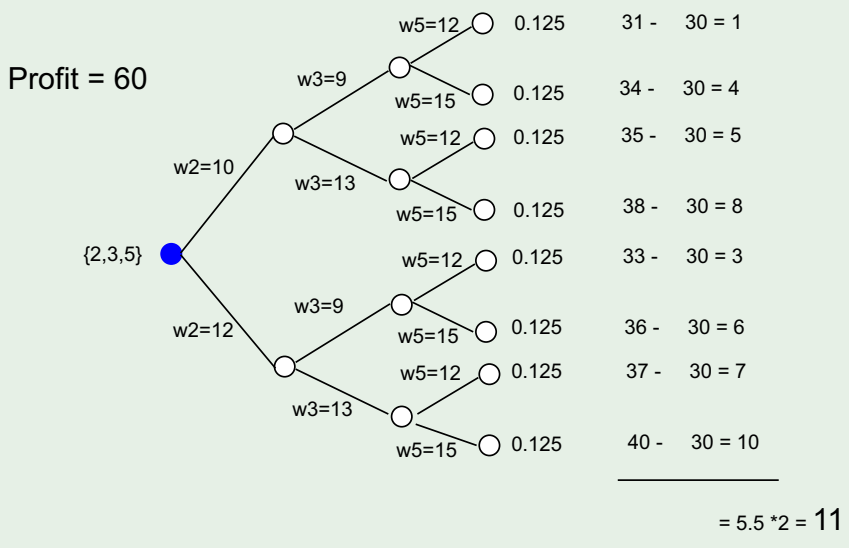
- This solution can be obtained by solving a **deterministic equivalent** conventional constraint program obtained by employing a **scenario-based** representation [6].



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- In Tarim et al. [6], the authors proposed a standard way of **compiling** down these models into **conventional** (non-stochastic) **CP models** that can be solved by any available commercial software.
- Of course this approach has **a price** since the number of scenarios that need to be considered in order to fully represent the problem **grows exponentially** with the number of decision stages in the problem.

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- Another possibility consists instead in developing **specialized and efficient filtering strategies**. For this purpose **global chance-constraints** have been proposed in Rossi et al. [4].
- In this work, by creating a parallel with Focacci et al. [2], we present **optimization-oriented global chance-constraints** as a way of enhancing the solving process of stochastic constraint programs.

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What differs is the fact that in a global optimization chance-constraint we find **two stages of relaxations**.

- At the **first stage** of relaxation we exploit known inequalities to replace stochastic variables in our stochastic programs with deterministic quantities and to yield a valid relaxation that is a deterministic problem.
- **A second stage of relaxation** may be needed to produce a deterministic problem that is computationally more tractable.

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- Define $\mathrm{P} = \max_{x \in S} z(x, \xi)$ as the **optimization problem** associated with **one particular scenario** $\xi \in \Xi$, where S is a *finite* set, and $z(x, \xi)$ is a real valued function of two (vector) variables x and ξ

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- Define $\mathrm{P} = \max_{x \in S} z(x, \xi)$ as the **optimization problem** associated with **one particular scenario** $\xi \in \Xi$, where S is a *finite* set, and $z(x, \xi)$ is a real valued function of two (vector) variables x and ξ

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Recourse Problem

- RP(P)$= \max_{x \in S} \sum_{\Xi} \Pr\{\xi\} z(x, \xi)$
- This can be expressed, by using the **expectation operator** $\mathbb{E}$, as $\mathrm{RP(P)} = \max_{x \in S} \mathbb{E} z(x, \Xi)$, with an optimal solution x^*

Value of Stochastic Solution

Expected Value Problem

The **deterministic problem** obtained by **replacing** all the stochastic (vector) variables by their **expected values**, is defined as

$$\mathrm{EV}(\mathrm{P}) = \max_{x \in S} z(x, \mathbb{E}[\Xi]).$$

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Let us denote by $\widehat{x}$ an optimal solution of the expected value problem, called the **expected value solution**. Anyone familiar with Stochastic Programming or realizing that uncertainty is a fact of life would feel **a little insecure about taking decision** $\widehat{x}$. Indeed, unless such a decision is independent of Ξ, there is **no reason to believe** that this decision is even **close to the optimal solution** of the recourse problem.

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It directly follows that EV(P) $\geq$ RP(P) (EV(P) $\leq$ RP(P)). We will base our **cost-based filtering strategies** on this **inequality**.

Expectation-based relaxation for stochastic variables

- The **first step** in our cost-based filtering strategy consists in applying a **relaxation involving the stochastic variables**.

Expectation-based relaxation for stochastic variables

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- By applying **Jensen's inequality** [EV(P) - RP(P) $\geq$ 0], if the profit (respectively cost for minimization problems) function satisfies the two assumptions discussed, an upper (lower) **bound** for the cost of an optimal solution to RP(P) can be **obtained by solving** EV(P), that is the **deterministic problem** in which all the stochastic variables are replaced by their respective **expected values**.

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The **profit function** for $RP(SSKP)$ is **concave** in $\overline{\mathcal{W}}$.

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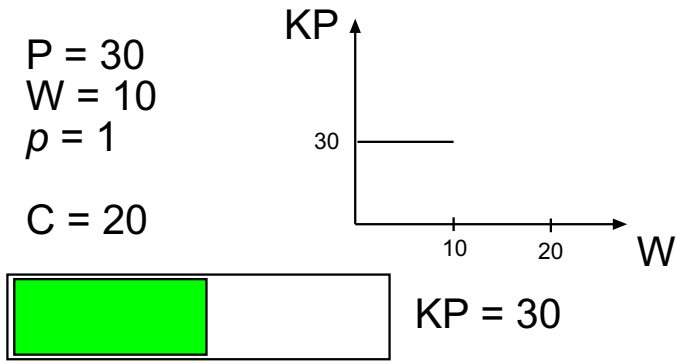

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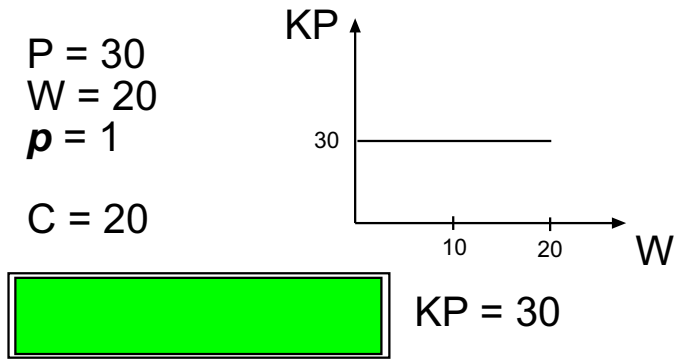

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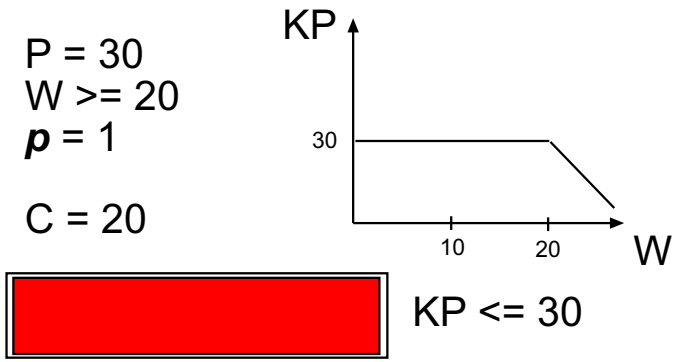

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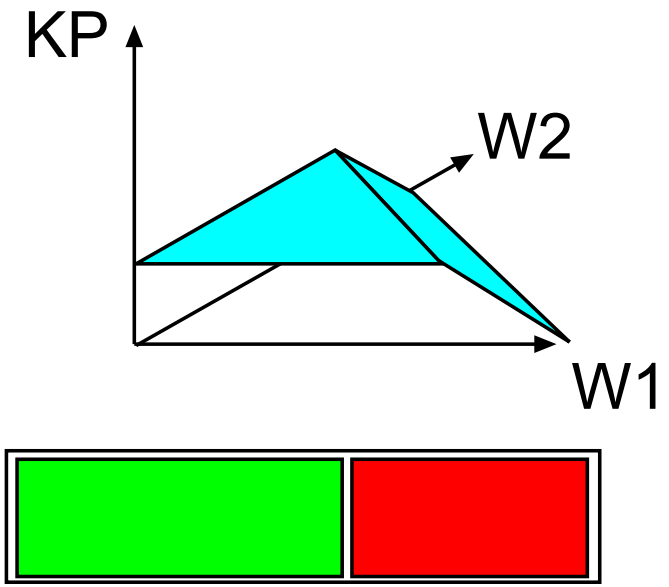

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- Obviously, in RP(SSKP), it is always possible to find a **feasible assignment** for decision variables, therefore **both the assumptions are satisfied** for this problem.
- The expected value problem EV(SSKP) can be obtained by **replacing every random variable** $\mathcal{W}_i$ in RP(SSKP) with the respective **expected value** $\mathbb{E}[\mathcal{W}_i]$, thus obtaining a fully deterministic model.

Expectation-based relaxation for stochastic variables

EV(SSKP)

Here we solve the **deterministic** problem where the weights of the objects are set to the respective **expected weights**: $\lfloor \mathbb{E}[f(1)] \rfloor = 9$, $\lfloor \mathbb{E}[f(2)] \rfloor = 11$, $\lfloor \mathbb{E}[f(3)] \rfloor = 11$, $\lfloor \mathbb{E}[f(4)] \rfloor = 5$, $\lfloor \mathbb{E}[f(5)] \rfloor = 13$.

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This problem provides the first **two pieces of information** needed by our cost-based filtering method

- (a) the optimal solution of the **relaxed problem**
- (b) the **optimal value of this solution**, which represents, according to Jensen's inequality, an **upper bound** for the original problem objective function. In our running example this solution selects items $3, 4, 5$ and has a value of EV(SSKP)$= 50$.

Relaxing the expected value problem

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- For this reason we can **further relax** the expected value problem in order to obtain a **valid bound** by solving an easier problem.
- Let REL(EV(P)) be a **generic relaxation** of EV(P). For a maximization problem EV(P)$\leq$ REL(EV(P)), by recalling that $R(P) \leq EV(P)$, REL(EV(P)) provides a valid bound for the recourse problem, $R(P)$.

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- In SSKP, for instance, **instead of solving** to optimality the deterministic (NP-Complete) knapsack problem obtained for the expected value scenario, we may instead **solve in linear time** its **continuous relaxation**, thus obtaining Dantzig's upper bound, DUB(EV(SSKP)).

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- DUB(EV(SSKP)) $\geq$ EV(SSKP) therefore DUB(EV(SSKP)) $\geq$ RP(SSKP).
- DUB(EV(SSKP)) is a **valid upper bound** for our recourse problem RP(SSKP).

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To obtain DUB(EV(SSKP))

- we **order items** for profit over expected weight: $\{25/13, 20/11, 15/11, 10/9, 5/5\}$

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Therefore DUB(EV(SSKP))$= 25 + 20 + (6 * 15/11) = 53.18$.

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- The bound obtained can be used to **exclude part of the tree** that cannot lead to a better solution.
- In [2] the authors discuss filtering strategies based on **reduced costs** (RC). As we shall see next a similar technique can be adopted for SSKP, provided that an efficient way of obtaining bounds is available for the expected value problem.

Cost-based filtering

- In order to perform **cost-based filtering**, as in RC-based filtering, we need a **gradient function grad(V,v)**, which returns for each variable-value pair (V,v) an optimistic evaluation of the profit obtained if v is assigned to V.

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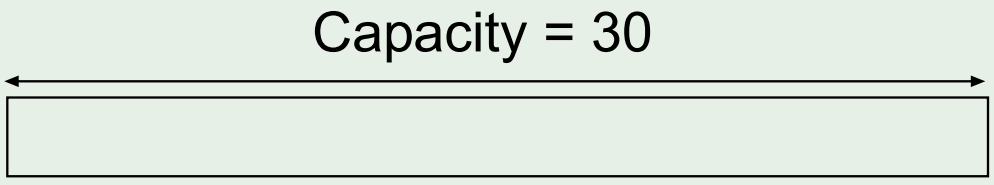
- This function is obviously **problem dependent**, but **regardless of the strategy adopted** in the former section — i.e. whether we are using a relaxation for the expected value problem or solving this problem to optimality — it is possible to specify it and use it to **filter provably suboptimal values**.

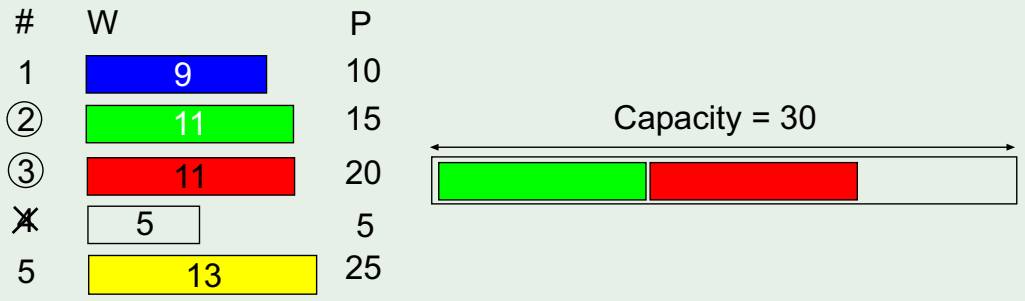
Cost-based Filtering for SSKP

#	W	P
1	9	10
2	11	15
3	11	20
4	5	5
5	13	25



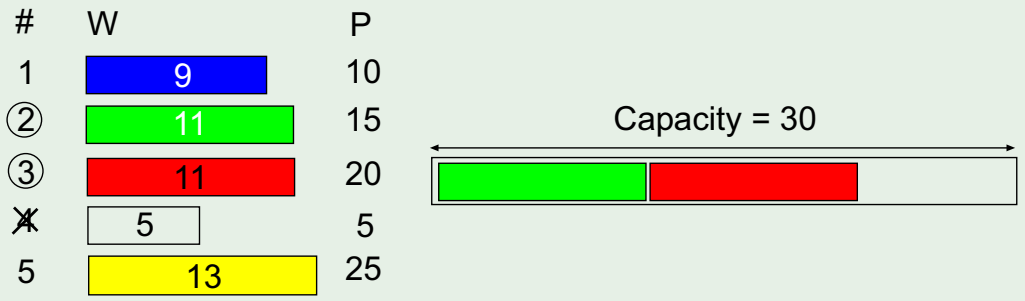
Cost-based Filtering for SSKP

We now consider the case in which items 2 and 3 have been **selected** in the knapsack and item 4 is **not selected**. We still have to **decide** on items 1 and 5.



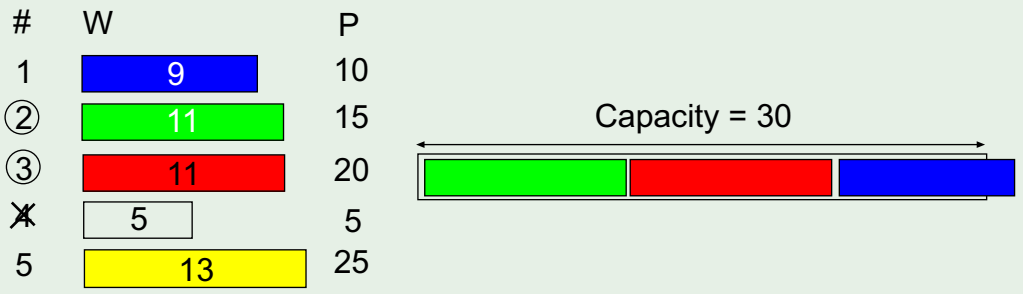
Cost-based Filtering for SSKP

We consider the set of the **remaining items** for which a decision must be taken, $K/S \equiv \{1, 5\}$.
Let us reason on item 1...



Cost-based Filtering for SSKP

Item 1 is a **critical** item, in fact if taken in the knapsack it will use more capacity than the residual $30 - 22 = 8$ units. If we consider the option of **taking this item**, then the expected profit is $\bar{r}_1 = 10 - 2 * (30 - 22 - 9) = 8$, there is no more residual capacity and item 5 is therefore excluded in the bound computation since $25 - 2 * 13 \leq 0$. The computed **bound** is $35 + 8 = 43$.



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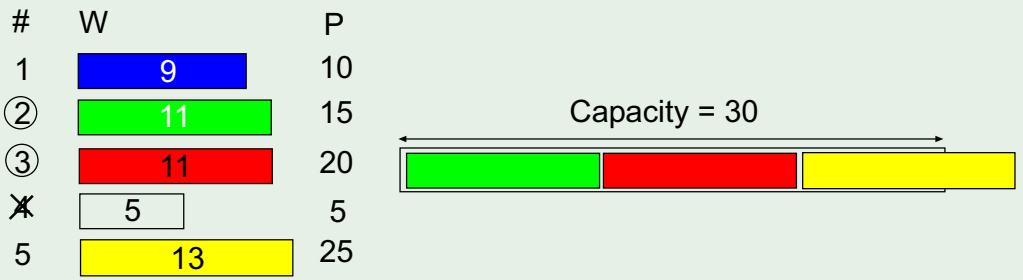
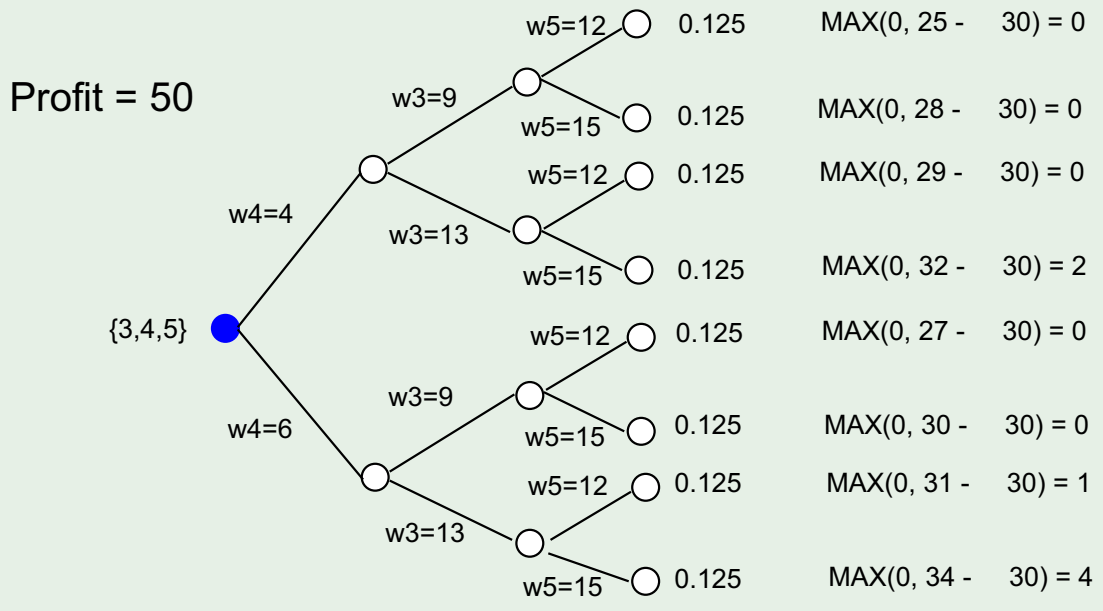
The reasoning is similar for item 5. If we consider the option of **taking this item**, then the expected profit is $\overline{r}_5 = 25 - 2 * (30 - 22 - 13) = 15$, there is **no more residual capacity** and item 1 is therefore **excluded** in the bound computation since $10 - 2 * 9 \leq 0$. The computed **bound** is $35 + 15 = 50$.



Cost-based Filtering for SSKP

Assume now that the current **best solution** for RP(SSKP) has a value of 48.25, corresponding to a knapsack that contains elements 3, 4 and 5: then element 1 can be excluded from the knapsack.



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Considerations

- Obviously, as discussed in [2] the information provided by the relaxed model (EV(P)), i.e. expected weights, gradient function etc., can be also used to **define search strategies**.

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- Obviously, as discussed in [2] the information provided by the relaxed model (EV(P)), i.e. expected weights, gradient function etc., can be also used to **define search strategies**.
- For instance in SSKP we may branch on variables according to a **decreasing profit over expected weight** heuristic, or selecting the one for which the **chosen gradient function gives the most promising value**.

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- **If such a solution is feasible** with respect to RP(P) — in our examples assumption (ii) guarantees this — we can easily compute EEV(P), that is *the expected result of using the EV(P) solution in the recourse problem RP(P)*.
- Furthermore, at every node of the search tree it is possible to adopt a **variable fixing strategy** and compute the EV(P) solution with respect to such a node.

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- This solution is **clearly feasible** for RP(SSKP).
- We can therefore compute EEV(SSKP)$=46$. This is, of course, a good **lower bound** for the objective function value.

Static Stochastic Knapsack Problem

Instance		Time		Nodes	
k	Scenarios	SCP	SCP-OO	SCP	SCP-OO
10	100	**0.4**	0.5	916	**100**
10	300	1.3	**0.5**	2630	**59**
10	500	2.4	**0.2**	4237	**8**
10	1000	7.2	**2.4**	6227	**120**
15	100	2.5	**0.3**	4577	**11**
15	300	15	**2.3**	10408	**252**
15	500	33	**1.1**	9982	**75**
15	1000	150	**6.3**	16957	**222**
20	100	70	**10**	102878	**1024**
20	300	250	**13**	85073	**953**
20	500	860	**9.5**	129715	**225**
20	1000	3200	**240**	134230	**7962**

Table: Experimental results for SSKP. Comparison between a pure SCP approach (SCP) and an SCP model enhanced with optimization-oriented global-chance constraints (SCP-OO), times are in seconds. In each line we indicated in bold the best performance in terms of run time and explored nodes.

Stochastic sequencing with release times and deadlines

Instance			Time		Nodes	
Jobs	Machines	Scenarios	SCP	SCP-OO	SCP	SCP-OO
3	2	10	**0.3**	**0.3**	203	**48**
3	2	30	1.3	**0.6**	701	**133**
3	2	50	3.2	**1.1**	927	**418**
3	2	100	12	**3.5**	1809	**838**
7	3	10	**180**	866	57688	**1723**
7	3	30	1800	**880**	186257	**5293**
7	3	50	3300	**1100**	212887	**6586**
7	3	100	14000	**1200**	277804	**8862**

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- In both cases these bounds are **not exploited** for **filtering** decision variable domains as in our approach and they cannot be used for **guiding the search**.

Conclusions

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- Our results confirm that **orders-of-magnitude improvements** in terms of **explored nodes** and **run times** can be achieved.

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- define strategies to **handle generic chance-constraints**
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Conclusions

In the future, we aim to

- apply cost-based filtering to **multi-stage** Stochastic COPs
- define strategies to **handle generic chance-constraints**
- extend the approach to **other valid inequalities** such as Edmundson-Madansky [1] or to suitable inequalities for non-convex problems [3]
- Finally, we plan to exploit the information provided by optimization-oriented global chance-constraints to define **search strategies**

The End

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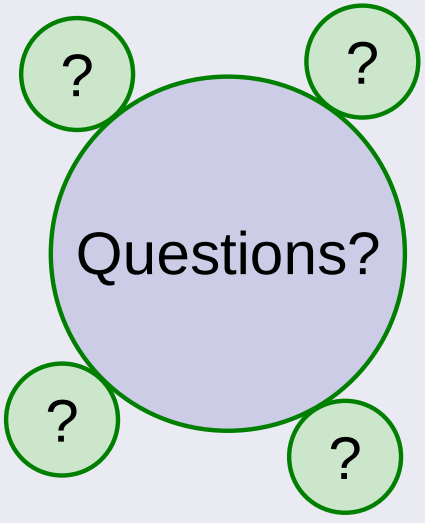

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