HACETTEPE UNIVERSITY

Cost-based Filtering for Stochastic Inventory Systems with Shortage Cost

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Cork Constraint Computation Centre

ctvr

ucc University College Cork

Inventory Control

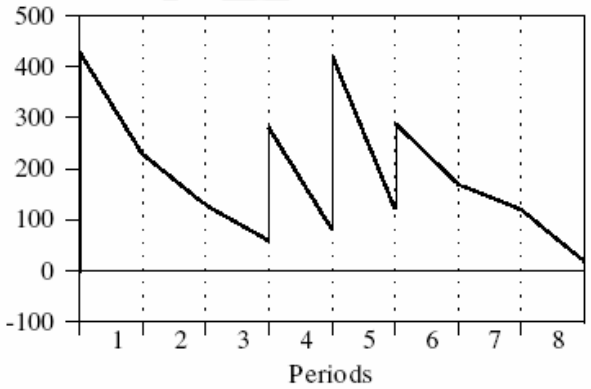
- Computation of optimal replenishment policies under demand uncertainty.

When to order?
How much to order?

?

Demand Uncertainty

Production

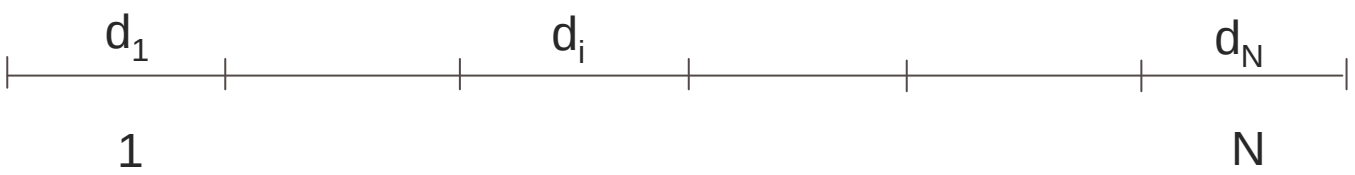


Customers

Unlimited capacity Deterministic Production Planning Problem [Wag – 58]

The unlimited capacity deterministic production/inventory control problem can be stated as follows:
INSTANCE Number $N \in \mathbb{Z}^+$ of periods, for each period i, $1 \leq i \leq N$, a demand $d_i \in \mathbb{Z}_0^+$. A production set-up cost $a_i \in \mathbb{Z}^+$, and an inventory cost coefficient $h_i \in \mathbb{Z}^+$, an overall bound $B \in \mathbb{Z}^+$.
QUESTION Do there exist production amounts $X_i \in \mathbb{Z}_0^+$, and associated inventory levels $I_i = \sum_{t=1}^{i}(X_t - d_t)$, $1 \leq i \leq N$, such that

$$\sum_{i=1}^{N}(h_i I_i) + \sum_{X_i>0} a_i \leq B\ ?$$



Cost components

Ordering cost
Holding cost

Problem parameters

Planning horizon length: N
Period demands: d_i

Unlimited capacity Deterministic Production Planning Problem [Wag – 58]

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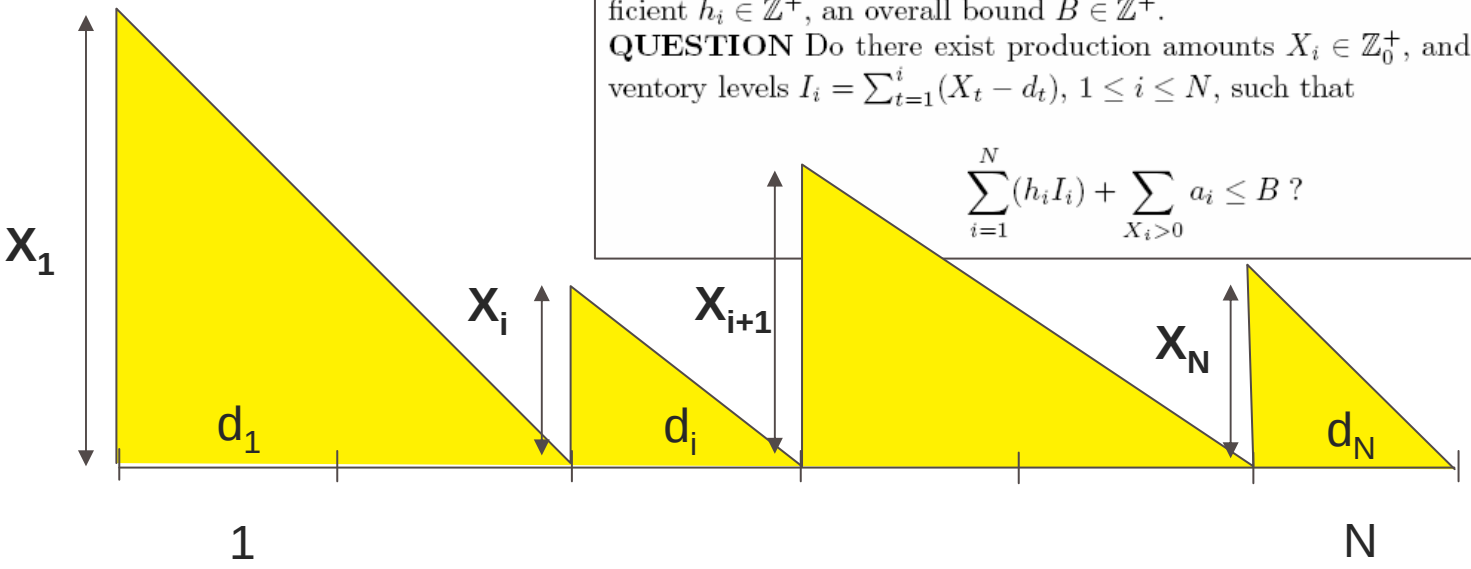

Cost components

Ordering cost
Holding cost

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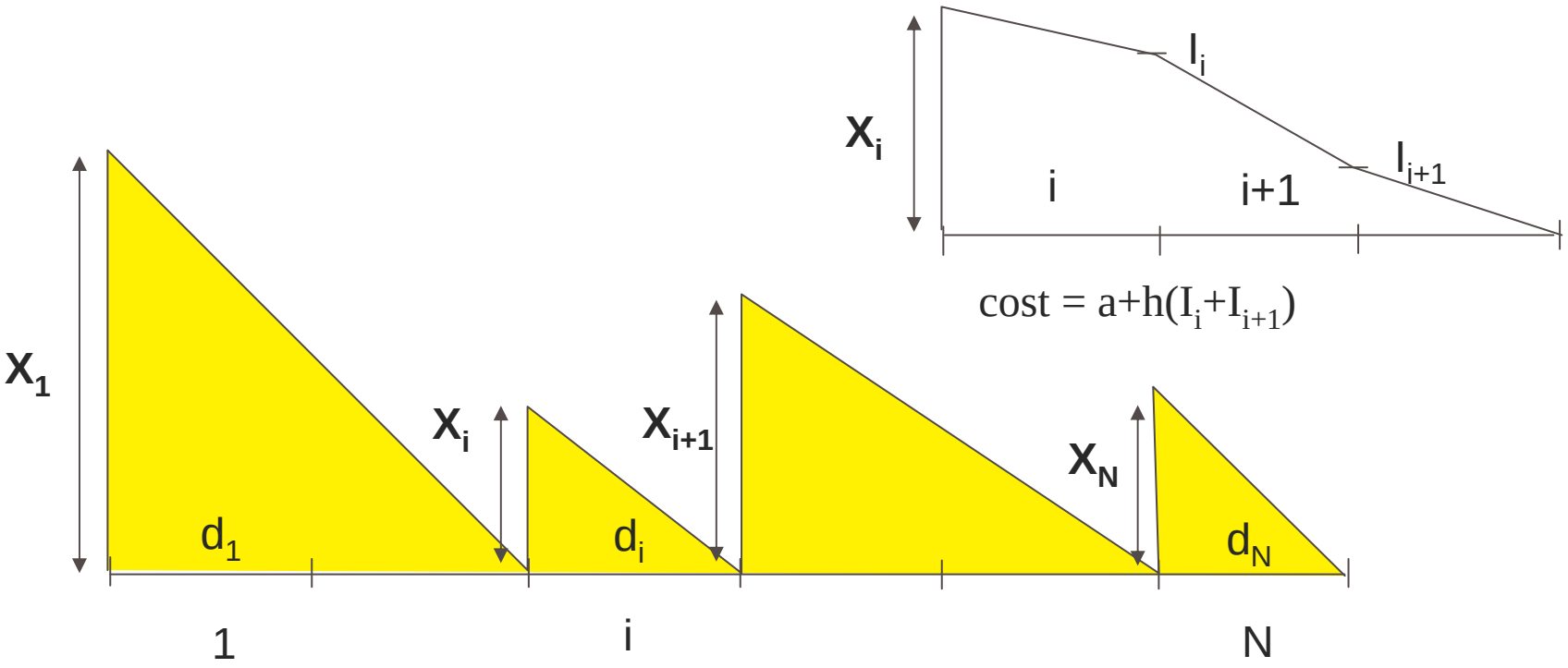
Unlimited capacity Deterministic Production Planning Problem [Wag – 58]



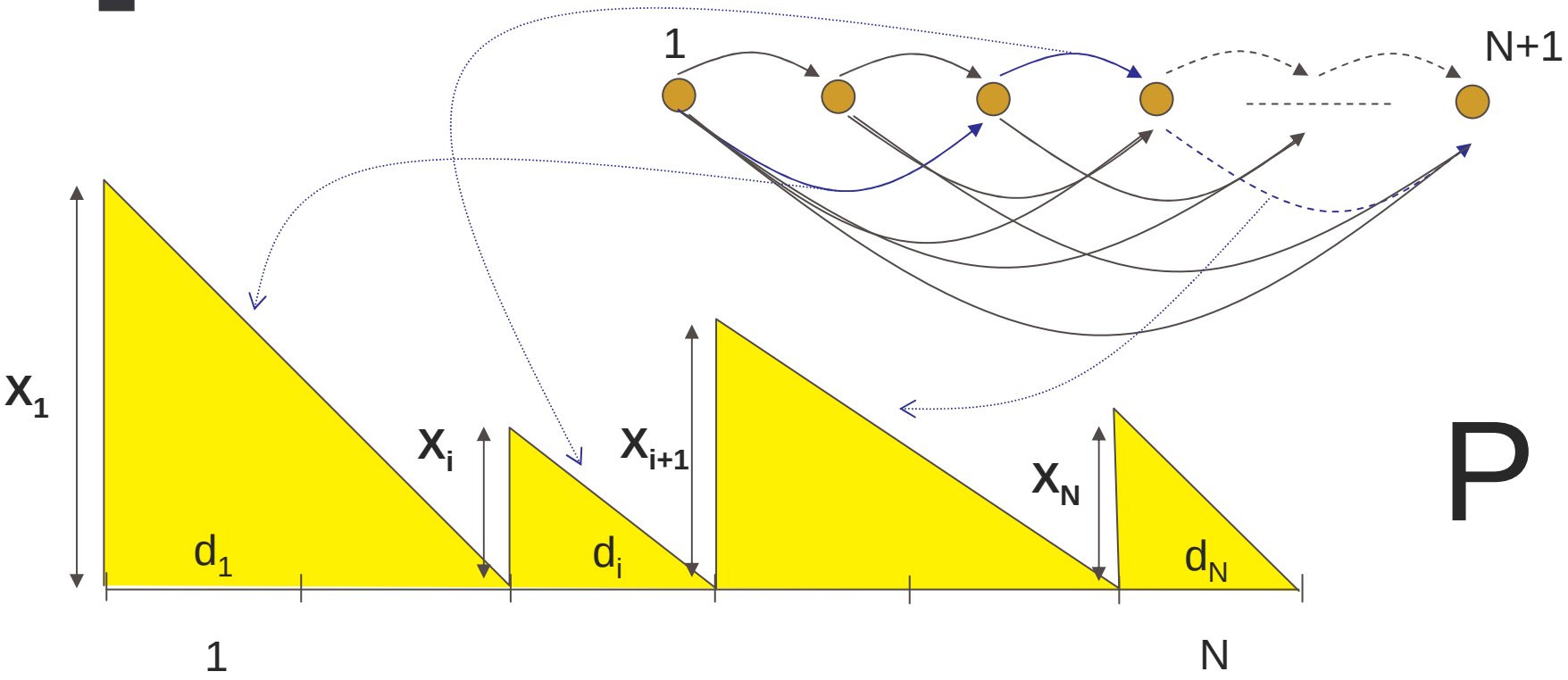
Cost components

Ordering cost	a
Holding cost	h

Problem parameters

Planning horizon length:	N
Period demands:	d_i

Unlimited capacity Deterministic Production Planning Problem [Wag – 58]



P

Cost components

Ordering cost
Holding cost

Problem parameters

Planning horizon length: N
Period demands: d_i

Capacitated Deterministic production planning [Gar – 79]

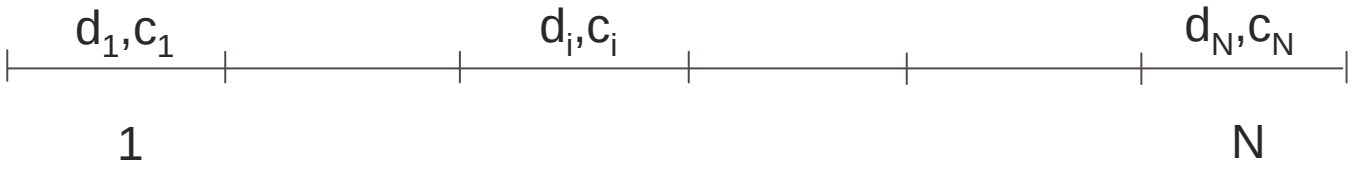
NP-hard

The capacitated deterministic production/inventory control problem can be stated as follows:

INSTANCE Number $N \in \mathbb{Z}^+$ of periods, for each period i, $1 \leq i \leq N$, a demand $d_i \in \mathbb{Z}_0^+$ and a production capacity $c_i \in \mathbb{Z}_0^+$. A production set-up cost $a_i \in \mathbb{Z}^+$, and an inventory cost coefficient $h_i \in \mathbb{Z}^+$, an overall bound $B \in \mathbb{Z}^+$.

QUESTION Do there exist production amounts $X_i \in \mathbb{Z}_0^+$ and associated inventory levels $I_i = \sum_{t=1}^{i}(X_t - d_t)$, $1 \leq i \leq N$, such that $X_i \leq c_i$, $1 \leq i \leq N$ and

$$\sum_{i=1}^{N}(h_i I_i) + \sum_{X_i>0} a_i \leq B\ ?$$



Cost components

Ordering cost
Holding cost

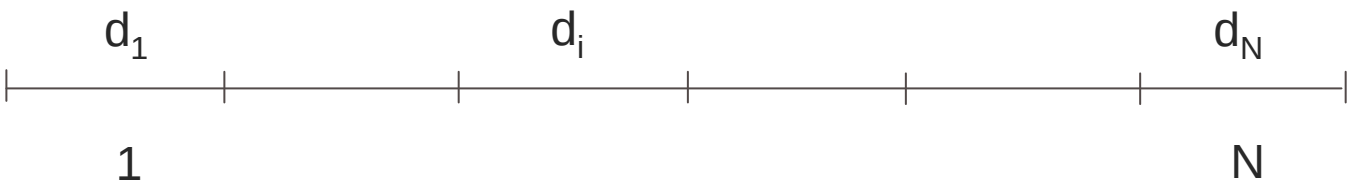
Problem parameters

Planning horizon length:	N
Period demands:	d_i
Production capacity:	c_i

Unlimited capacity Stochastic Production Planning

The unlimited capacity stochastic production/inventory control problem can be stated as follows:
INSTANCE Number $N \in \mathbb{Z}^+$ of periods, for each period i, $1 \leq i \leq N$, a random demand $d_i \in \mathbb{Z}_0^+$. A production set-up cost $a_i \in \mathbb{Z}^+$, an inventory cost coefficient $h_i \in \mathbb{Z}^+$, a shortage cost coefficient $p_i \in \mathbb{Z}^+$, an overall bound $B \in \mathbb{Z}^+$.
QUESTION Do there exist production amounts $X_i \in \mathbb{Z}_0^+$ and associated inventory levels $I_i = \sum_{t=1}^{i}(X_t - d_t)$, $1 \leq i \leq N$, such that

$$\sum_{i=1}^{N}(h_i I_i^+) + \sum_{i=1}^{N}(p_i I_i^-) + \sum_{X_i>0} a_i \leq B \ ?$$



Cost components

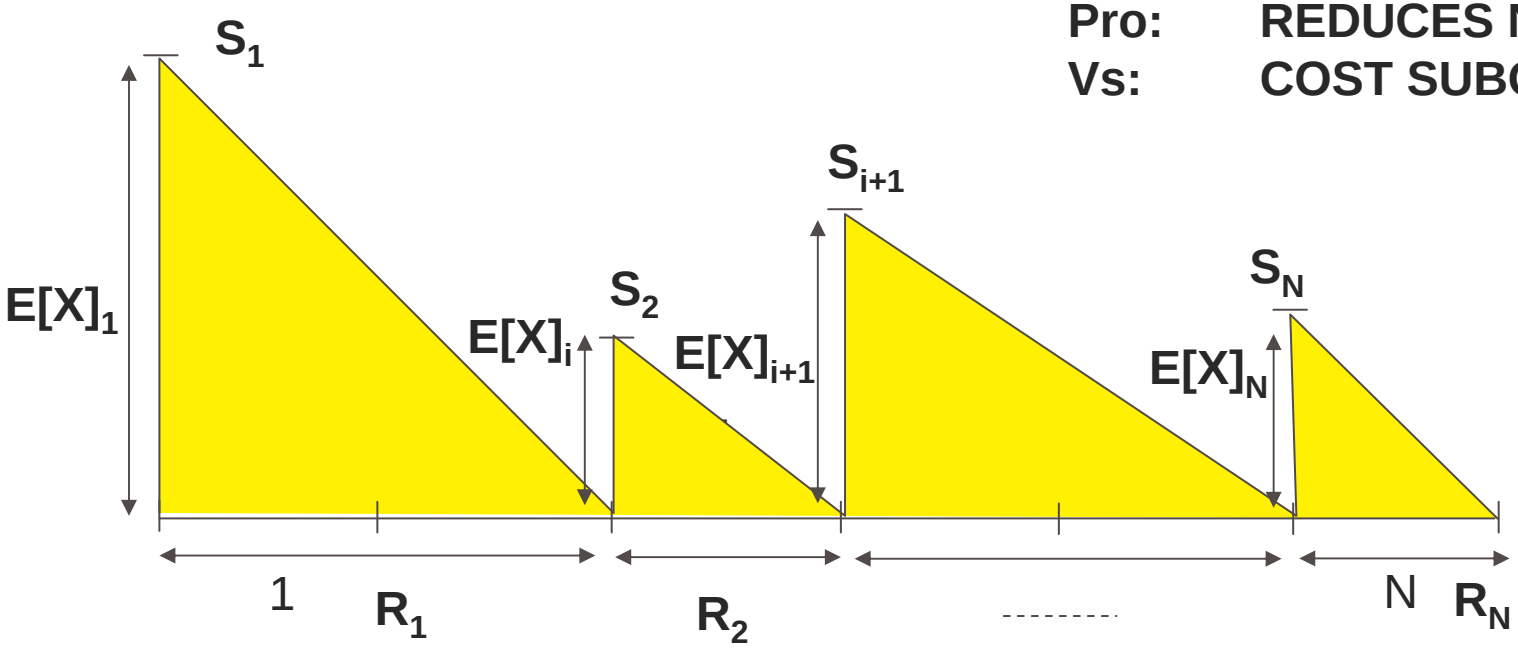
Ordering cost
Holding cost
Shortage cost

Problem parameters

Planning horizon length: N
Coefficient of variation: c_v
Expected demands: d_i

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

Pro: **REDUCES NERVOUSNESS**
Vs: **COST SUBOPTIMAL**



Cost components

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Holding cost
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Problem parameters

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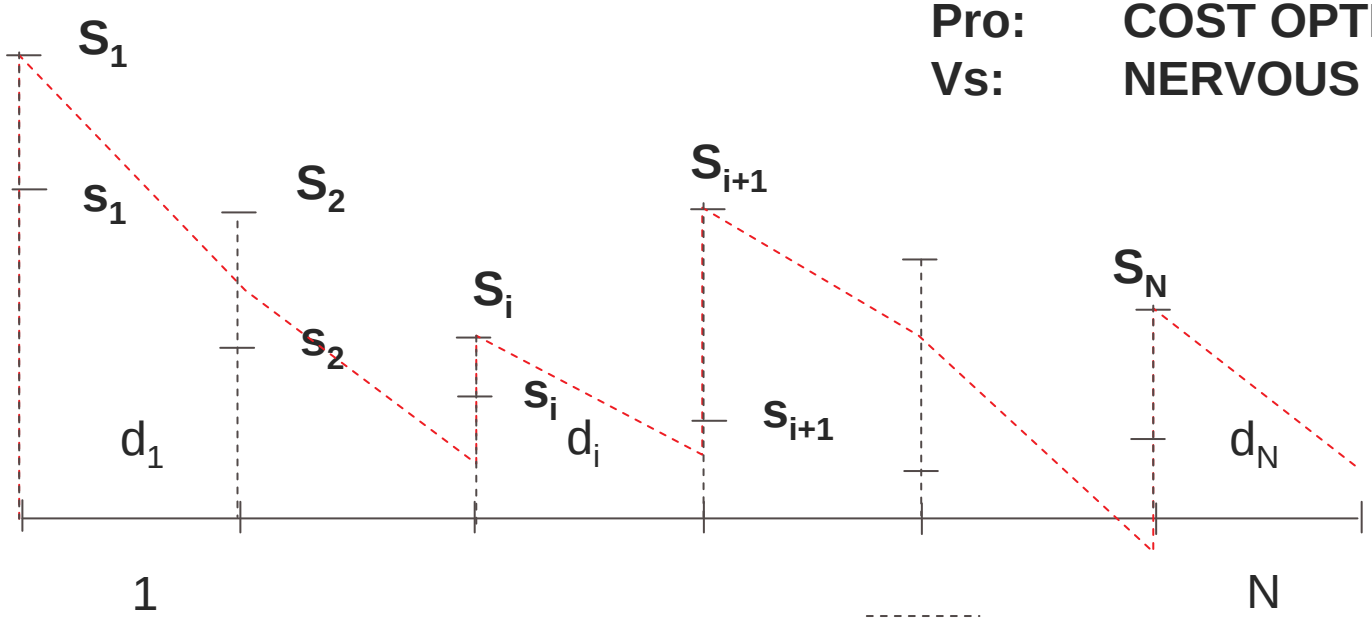
Unlimited capacity Stochastic Production Planning – (S^n,s^n) Policy

Pro: **COST OPTIMAL**
Vs: **NERVOUS POLICY**



Cost components

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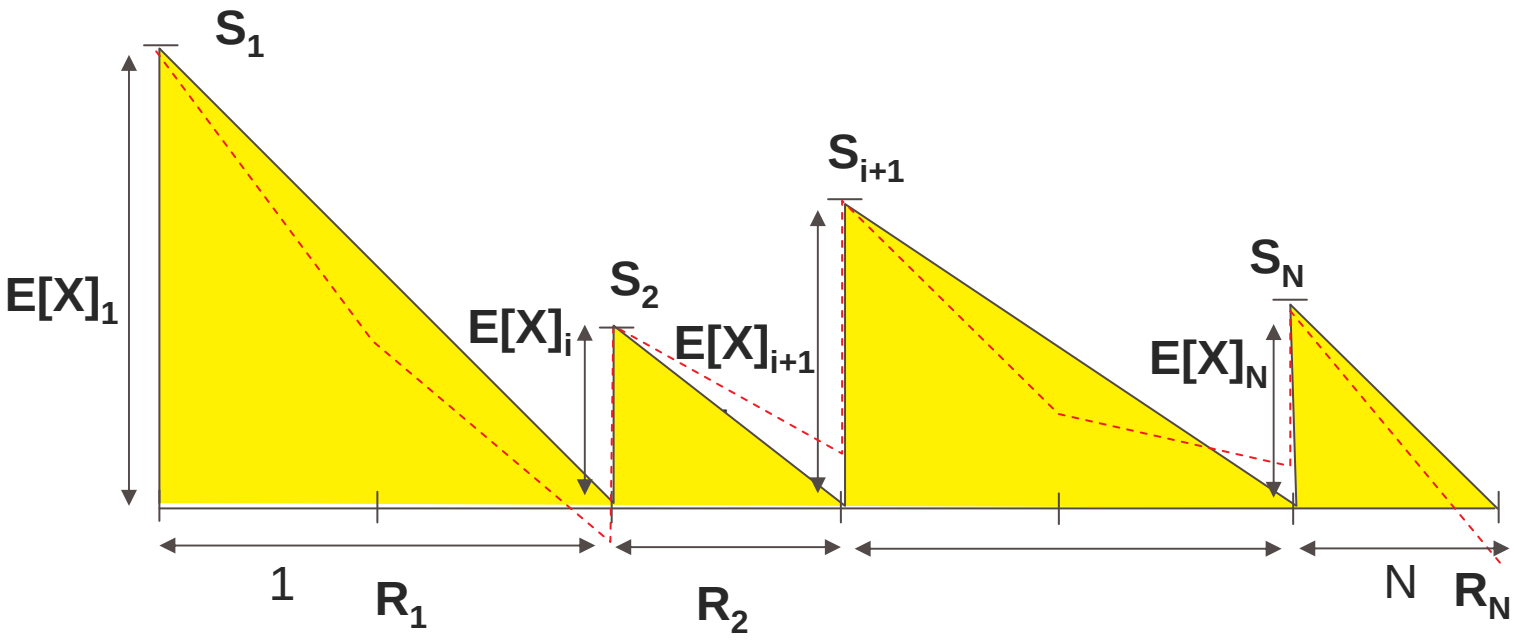
Planning horizon length: N
Coefficient of variation: c_v
Expected demands: d_i

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- Replenishment cycle policy (R,S)
  - effective in damping *planning instability,* also known as *control nervousness.*
  - Silver [Sil – 98] points out that this policy is appealing in several cases:
    - Items ordered from the same supplier (joint replenishments)
    - Items with resource sharing
    - Workload prediction
    - *...*
- Dynamic (R,S), heuristic approach [Boo – 88]
  - Considers a non-stationary demand over an N-period planning horizon

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy



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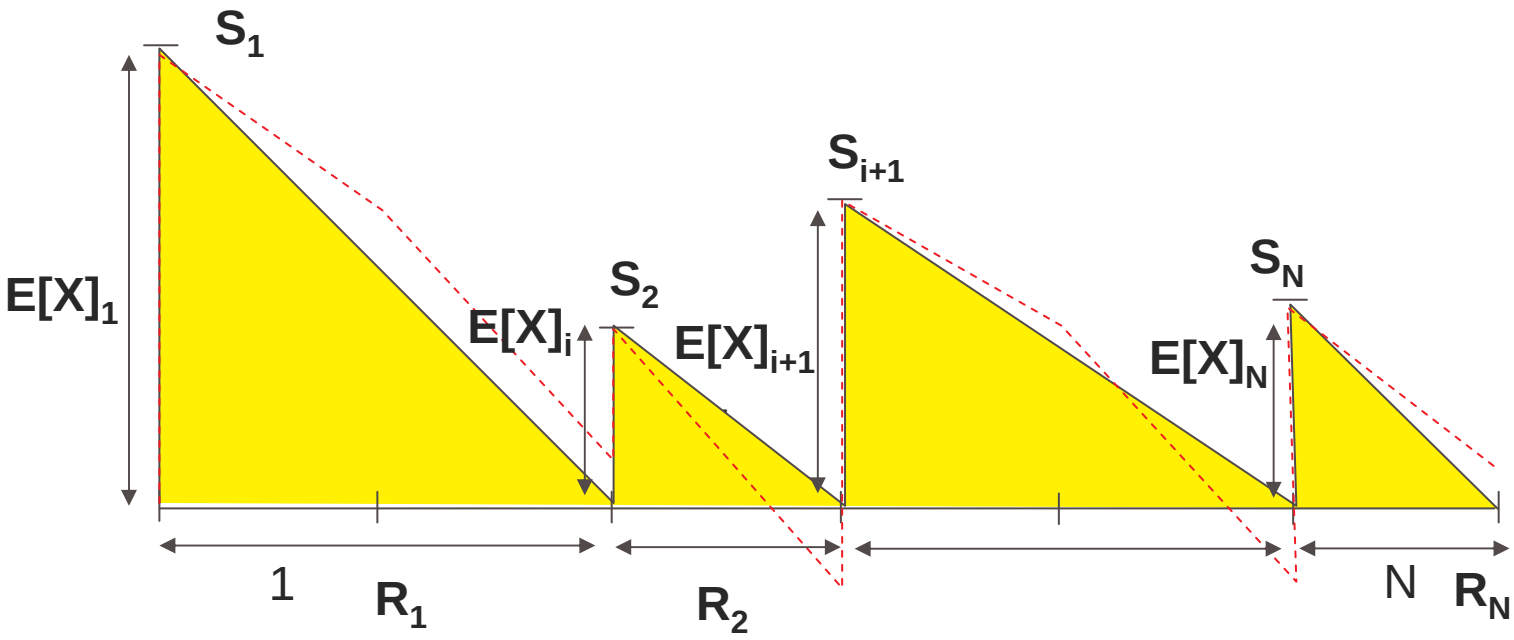

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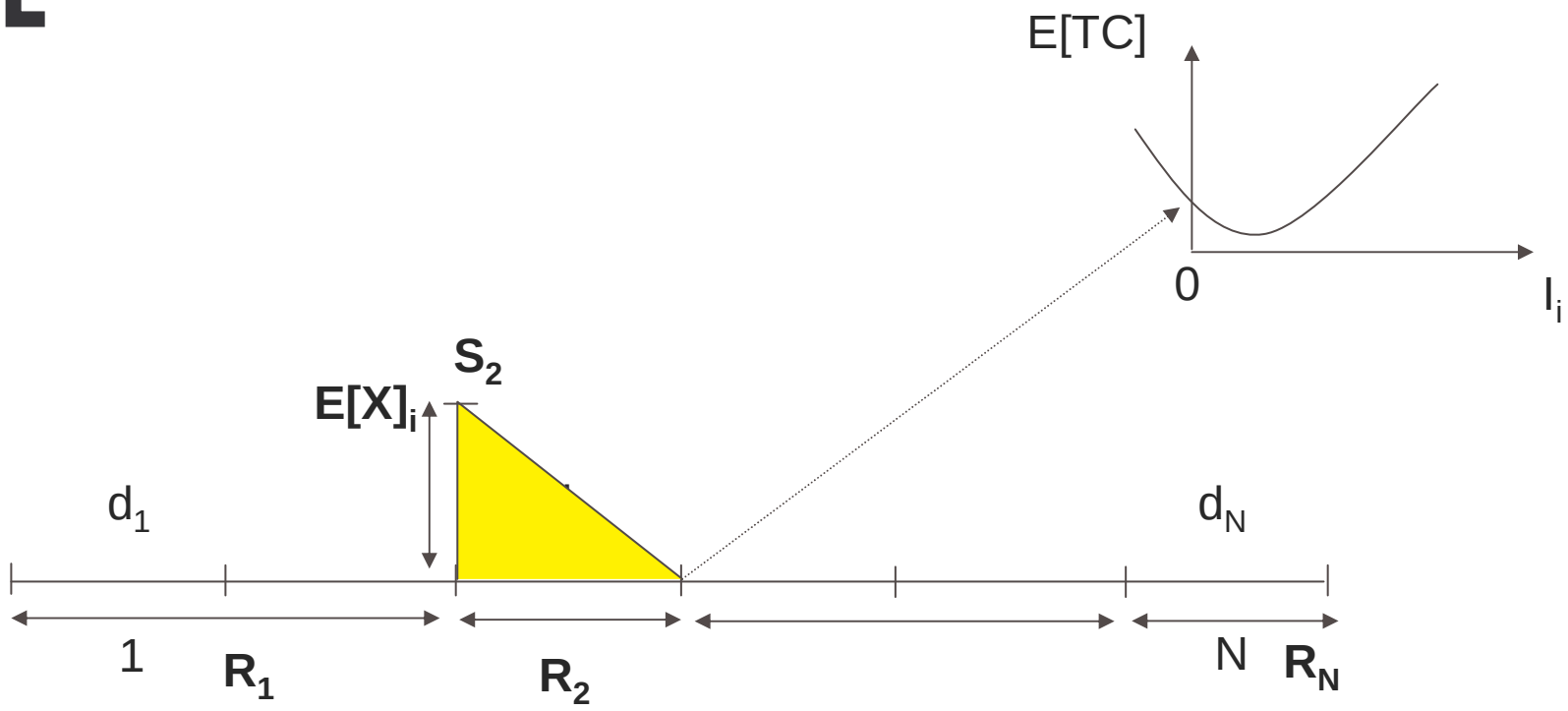
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy



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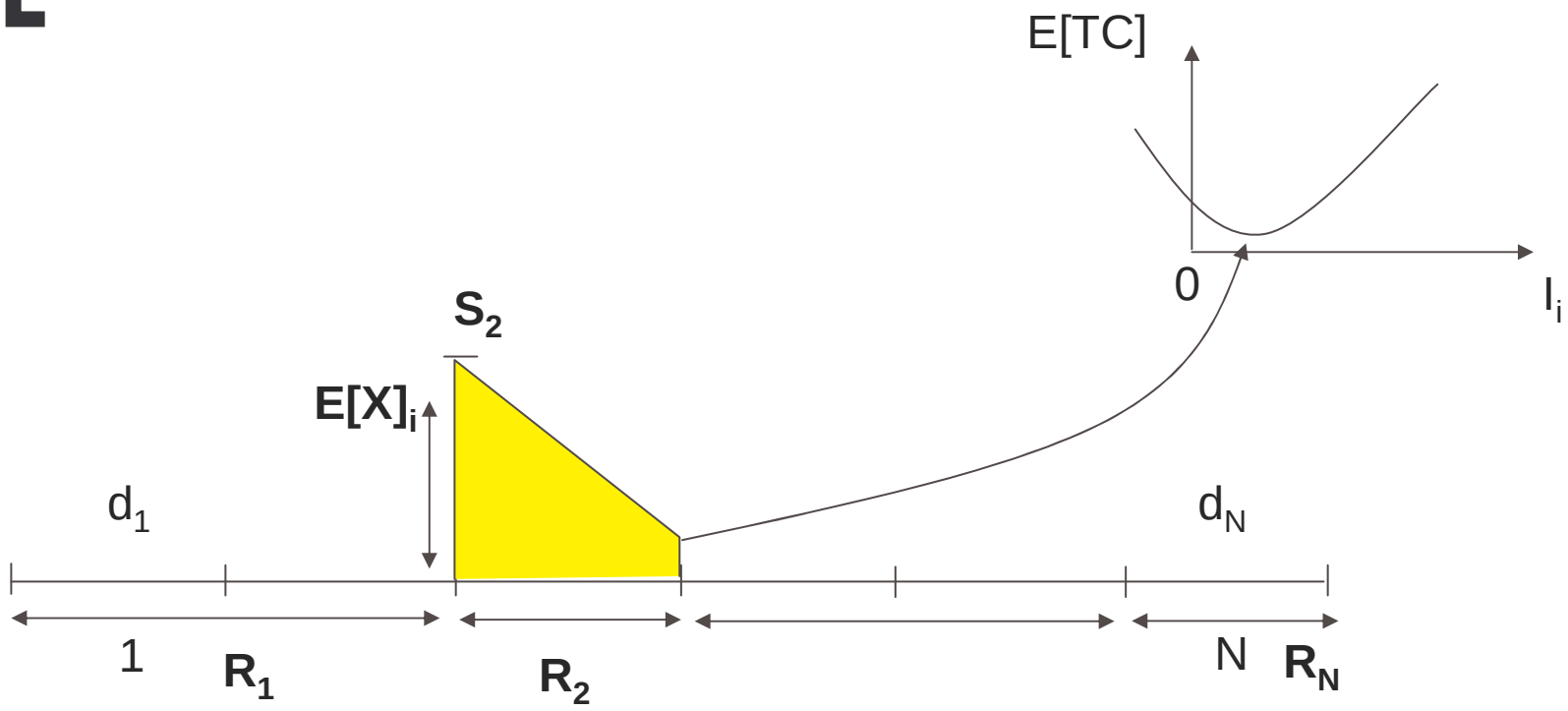
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy



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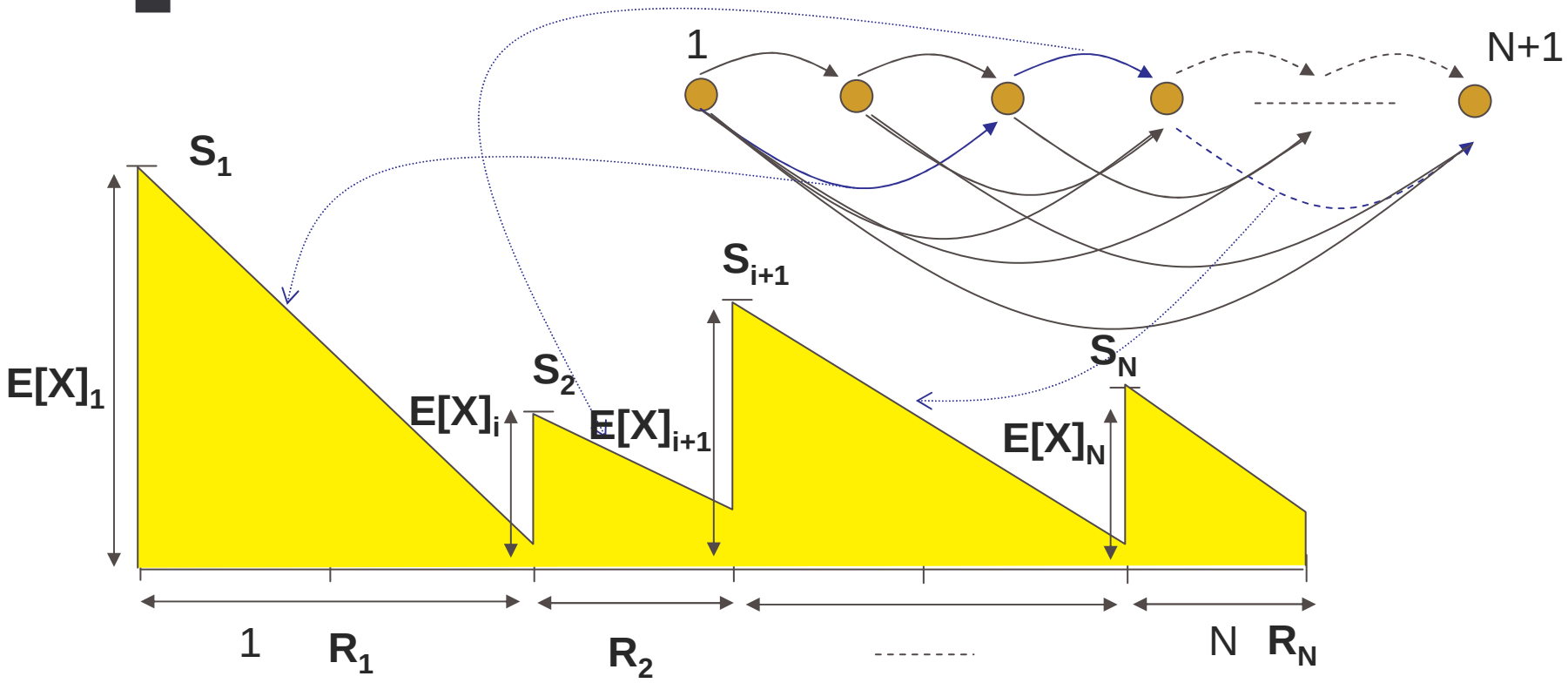
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy



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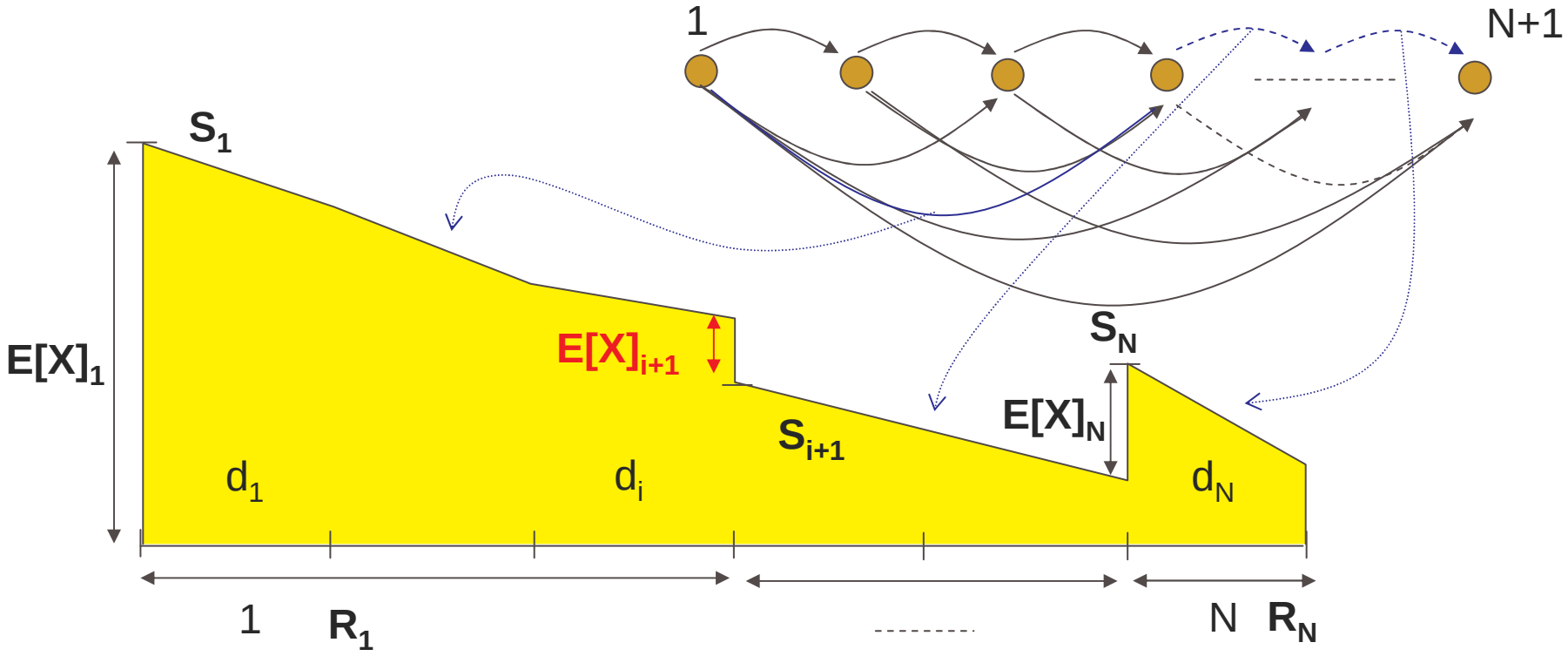
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy



Cost components

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Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- A *deterministic equivalent* [Bir – 97] CP formulation is [RTHP – 07]:

$$\min \; E\{TC\} = C \tag{18}$$

subject to

$$\text{objConstraint}\left(C, \tilde{I}_1, \ldots, \tilde{I}_N, \delta_1, \ldots, \delta_N, d_1, \ldots, d_N, a, h, s\right) \tag{19}$$

and for $t = 1 \ldots N$

$$\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} \geq 0 \tag{20}$$

$$\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} > 0 \Rightarrow \delta_t = 1 \tag{21}$$

$$\tilde{I}_t \in \mathbf{Z}, \quad \delta_t \in \{0, 1\} \tag{22}$$

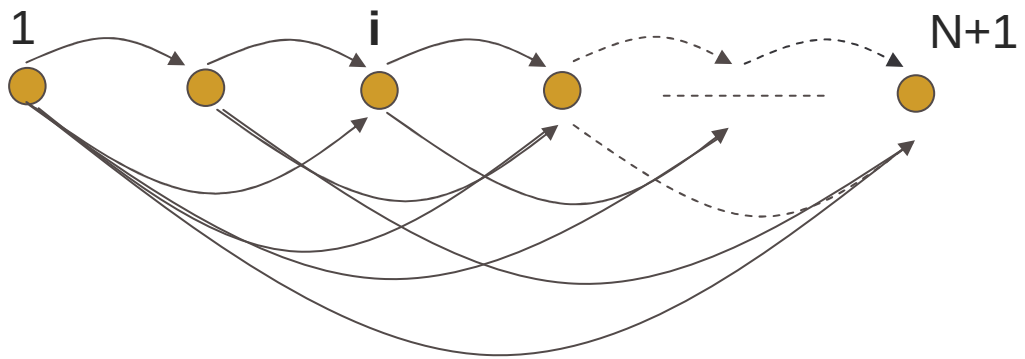
The objective function (18) minimizes the expected total cost over the given planning horizon. $objConstraint(\cdot)$ dynamically computes buffer stocks and it assigns to C the expected total cost related to a given assignment for replenishment decisions, depending on the demand distribution in each period and on the given combination for problem parameters a, h, s.

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- Dedicated cost-based filtering techniques (see [Foc – 99]) can be developed
- In [Tar – 07] we already presented a similar filtering method under a service level constraint [Tar – 05, Tar – 04].
  - Dynamic programming relaxation [Tar – 96].
- This technique lets us solve instances with planning horizons up to 50 periods typically in less than a second for the service level case [Tar – 07]

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

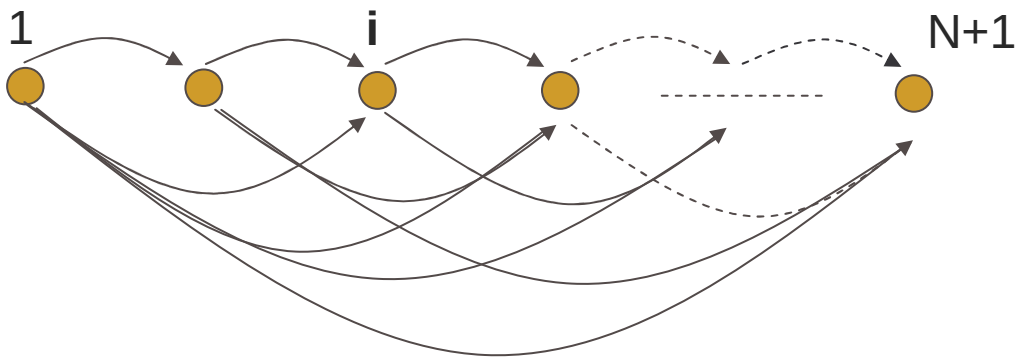
- **Cost-based filtering by relaxation**: the nodes in the shortest path are the optimal replenishment periods.



- (i) negative orders are scheduled (infeasible policy)
- (ii) no negative order arises (feasible and optimal policy)

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- **Cost-based filtering by relaxation**:
- (a) **i** is not a replenishment period



Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

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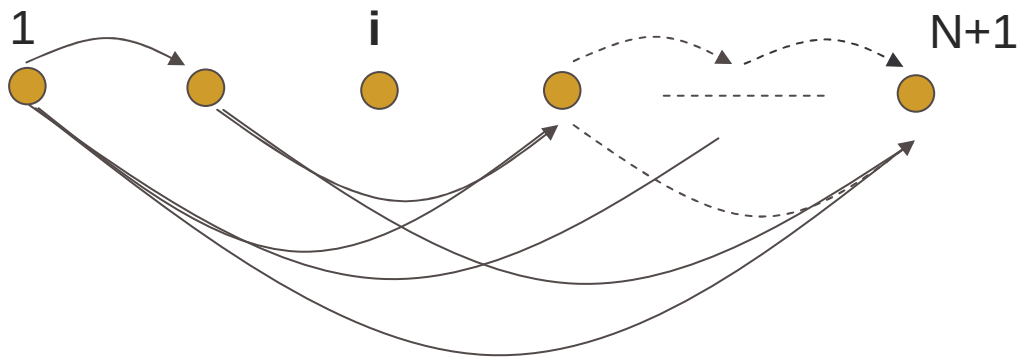
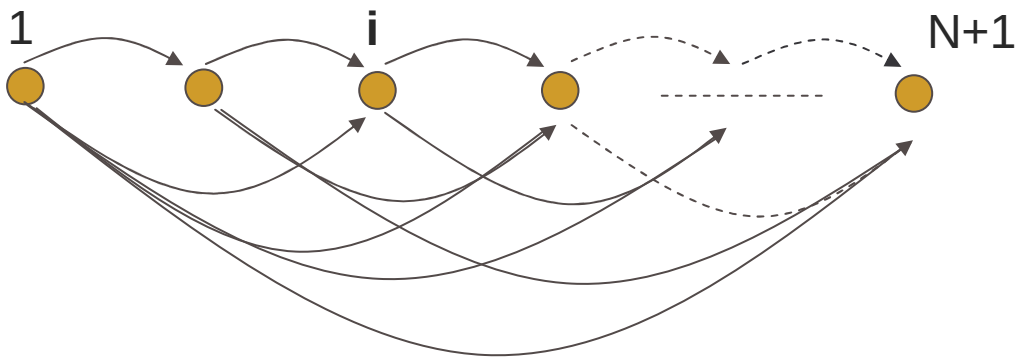


- We remove from the network every inbound (and outbound) arc to (from) node **i**. By doing this we will look for a shortest path that is not passing through node **i**.

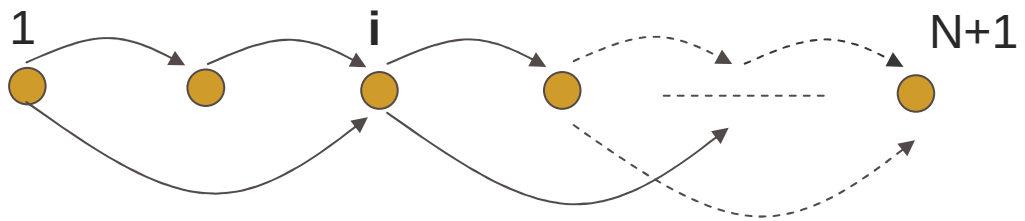
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- **Cost-based filtering by relaxation**:
- (b) **i** is a replenishment period



Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

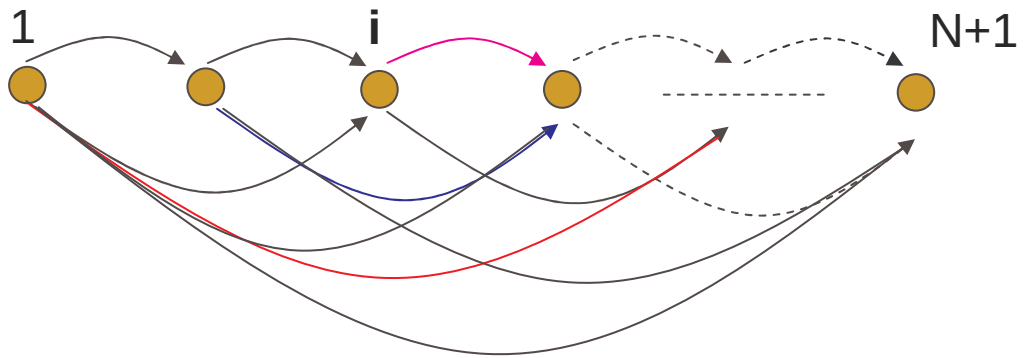
- **Cost-based filtering by relaxation**:
- (b) **i** is a replenishment period



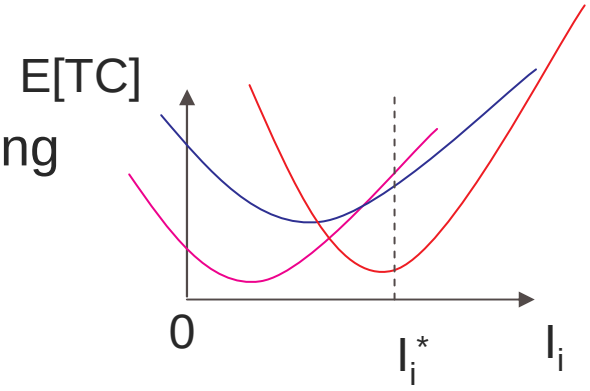
- We remove from the network every arc (h,j), where h<i<j. By doing this we will look for a shortest path that is passing through node **i**.

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- **Cost-based filtering by relaxation**:
- (c) the expected closing inventory level at the end of period **i** has been assigned to I_i^*



- The expected total cost of cycles covering period i is fixed



Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy Experimental results

		Test Set P1				Test Set P2			
		$\sigma_t/\bar{d}_t = 1/3$		$\sigma_t/\bar{d}_t = 1/6$		$\sigma_t/\bar{d}_t = 1/3$		$\sigma_t/\bar{d}_t = 1/6$	
a	N	$s = 15$	$s = 25$	$s = 15$	$s = 25$	$s = 15$	$s = 25$	$s = 15$	$s = 25$
50	20	0,150	0,030	0,020	0,020	0,030	0,040	0,050	0,050
	22	−−	−−	0,020	0,030	0,040	0,030	0,060	0,060
	24	0,031	0,040	0,030	0,030	0,060	0,040	0,080	0,070
	26	0,040	0,070	0,040	0,040	0,060	0,050	0,120	0,120
	28	0,050	0,080	0,060	0,050	0,070	0,060	0,170	0,121
	30	0,080	0,090	0,060	0,050	0,080	0,081	0,161	0,161
	32	0,100	0,090	0,070	0,081	0,120	0,141	0,180	0,150
	34	−−	−−	0,060	0,070	0,140	0,080	0,180	0,160
	36	0,210	0,111	0,080	0,081	0,161	0,090	0,230	0,180
	38	0,171	0,100	0,090	0,080	0,140	0,120	0,210	0,241
100	20	0,030	5,949	0,020	0,030	0,040	0,030	0,020	0,020
	22	0,030	0,030	0,030	0,030	0,040	0,030	0,031	0,030
	24	0,030	0,040	0,040	0,030	0,040	0,041	0,040	0,030
	26	0,040	0,040	0,040	0,040	0,080	0,050	0,050	0,050
	28	0,060	0,070	0,050	0,050	0,060	0,071	0,060	0,051
	30	0,061	0,060	0,060	0,060	0,071	0,080	0,061	0,080
	32	0,080	−−	0,070	0,070	0,081	0,090	0,071	0,070
	34	0,070	0,060	0,070	0,070	0,090	0,080	0,231	0,070
	36	0,080	0,101	0,071	0,071	0,101	0,100	0,090	0,090
	38	0,080	0,101	0,090	0,091	0,110	0,120	0,100	0,101
150	20	0,020	0,020	0,030	0,021	0,030	0,020	0,020	0,030
	22	0,030	0,030	0,030	0,020	0,030	0,030	0,030	0,030
	24	0,040	0,040	0,030	0,030	0,040	0,040	0,040	0,030
	26	0,040	0,040	0,040	0,040	0,040	0,050	0,050	0,061
	28	0,050	0,050	0,050	0,041	0,060	0,061	0,050	0,050
	30	0,070	0,071	0,050	0,061	0,070	0,070	0,060	0,070
	32	0,070	4,306	0,060	0,071	0,080	0,080	0,070	0,070
	34	0,070	0,070	0,060	0,070	0,100	0,080	0,070	0,071
	36	0,080	0,080	0,070	0,080	0,090	0,110	0,080	0,090
	38	0,090	0,100	0,100	0,080	0,110	0,120	0,110	0,121
200	20	0,030	0,030	0,030	0,020	0,031	0,040	0,030	0,020
	22	0,030	0,220	0,030	0,030	0,030	0,041	0,030	0,030
	24	0,030	0,040	0,030	0,040	0,040	0,040	0,030	0,041
	26	0,040	0,040	0,040	0,040	0,050	0,051	0,041	0,050
	28	0,050	0,050	0,051	0,060	0,080	0,060	0,060	0,050
	30	0,070	0,060	0,060	0,060	0,070	0,070	0,070	0,070
	32	0,080	0,080	0,060	0,060	0,080	0,090	0,070	0,070
	34	0,070	−−	0,070	0,070	0,090	0,080	0,080	0,081
	36	0,080	0,081	0,070	0,070	0,110	0,101	0,090	0,110
	38	0,100	0,090	0,091	0,090	0,121	0,100	0,110	0,110

Table 4
Test Set P1, P2.

		Test Set P3				Test Set P4			
		$\sigma_t/\bar{d}_t = 1/3$		$\sigma_t/\bar{d}_t = 1/6$		$\sigma_t/\bar{d}_t = 1/3$		$\sigma_t/\bar{d}_t = 1/6$	
a	N	$s = 15$	$s = 25$	$s = 15$	$s = 25$	$s = 15$	$s = 25$	$s = 15$	$s = 25$
50	20	0,321	0,170	0,330	0,160	0,070	0,030	0,050	0,061
	22	0,480	0,300	0,370	0,341	0,030	0,040	0,060	0,060
	24	0,581	0,310	0,531	0,421	0,050	0,040	0,110	0,071
	26	1,222	0,501	0,791	0,531	0,070	0,060	0,090	0,090
	28	2,224	0,661	1,142	0,741	0,140	0,070	0,120	0,160
	30	2,013	0,722	1,052	0,751	0,100	0,060	0,130	0,170
	32	1,812	0,941	1,182	0,801	0,121	0,080	0,180	0,140
	34	1,883	0,862	1,312	0,952	0,120	0,090	0,190	0,150
	36	2,093	0,981	1,472	1,152	0,121	0,110	0,210	0,180
	38	3,636	1,131	1,803	1,512	0,120	0,100	0,251	0,200
100	20	0,030	0,040	0,060	0,070	0,040	0,030	0,030	0,020
	22	0,040	0,040	0,070	0,071	0,040	0,030	0,030	0,030
	24	0,040	0,050	0,090	0,080	0,050	0,030	0,040	0,040
	26	0,050	0,281	0,100	0,100	0,050	0,050	0,050	0,040
	28	0,070	0,070	0,131	0,120	0,061	0,060	0,060	0,060
	30	0,070	0,070	0,140	0,130	0,070	0,070	0,070	0,060
	32	0,080	0,080	0,150	0,160	0,080	0,080	0,070	0,070
	34	0,090	0,090	0,161	0,210	0,090	0,081	0,080	0,070
	36	0,100	0,110	0,240	0,180	0,090	0,090	0,090	0,080
	38	0,141	0,130	0,211	0,250	0,100	0,100	0,110	0,100
150	20	0,040	0,030	0,060	0,060	0,030	0,030	0,030	0,030
	22	0,040	0,040	0,071	0,070	0,030	0,030	0,030	0,030
	24	0,050	0,041	0,140	0,080	0,040	0,030	0,040	0,030
	26	0,060	0,050	0,100	0,160	0,060	0,040	0,050	0,040
	28	0,070	0,070	0,120	0,170	0,060	0,060	0,060	0,050
	30	0,070	0,070	0,130	0,140	0,070	0,060	0,070	0,060
	32	0,090	0,090	0,160	0,220	0,080	0,080	0,070	0,070
	34	0,091	0,090	0,171	0,170	0,080	0,090	0,080	0,080
	36	0,100	0,110	0,181	0,250	0,100	0,100	0,090	0,090
	38	0,140	0,120	0,220	0,260	0,100	0,101	0,100	0,100
200	20	0,071	0,030	0,060	0,070	0,040	0,030	0,030	0,030
	22	0,090	0,070	0,070	0,120	0,030	0,030	0,030	0,030
	24	0,090	0,130	0,080	0,080	0,050	0,040	0,030	0,040
	26	0,110	0,110	0,100	0,171	0,050	0,050	0,051	0,050
	28	0,130	0,170	0,181	0,130	0,070	0,070	0,060	0,050
	30	0,210	0,150	0,150	0,151	0,060	0,070	0,070	0,060
	32	0,210	0,090	0,150	0,221	0,070	0,070	0,070	0,080
	34	0,210	0,241	0,180	0,160	0,080	0,080	0,080	0,081
	36	0,250	0,210	0,241	0,190	0,090	0,100	0,080	0,090
	38	0,221	0,271	0,260	0,210	0,140	0,110	0,100	0,131

Table 5
Test Set P3, P4.

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy Conclusions

- We presented a **cost based filtering technique** for a CP approach that finds optimal (R^n,S^n) policies under nonstationary demands and a shortage cost scheme

- Our computational experiments prove the effectiveness of our approach

Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy References

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