Cost-based Filtering for Stochastic Inventory Systems with Shortage Cost

**Student:**
Roberto Rossi
UCC, Ireland

**Supervisors:**
S. Armagan Tarim
HU, Turkey
Brahim Hnich
IUE, Turkey
Steven Prestwich
UCC, Ireland
Inventory Control

- Computation of optimal replenishment policies under demand uncertainty.

When to order? How much to order?

Demand Uncertainty

Production

Customers
The unlimited capacity deterministic production/inventory control problem can be stated as follows:

**INSTANCE** Number $N \in \mathbb{Z}^+$ of periods, for each period $i$, $1 \leq i \leq N$, a demand $d_i \in \mathbb{Z}_0^+$. A production set-up cost $a_i \in \mathbb{Z}^+$, and an inventory cost coefficient $h_i \in \mathbb{Z}^+$, an overall bound $B \in \mathbb{Z}^+$.

**QUESTION** Do there exist production amounts $X_i \in \mathbb{Z}_0^+$, and associated inventory levels $I_i = \sum_{t=1}^{i}(X_t - d_t)$, $1 \leq i \leq N$, such that

$$\sum_{i=1}^{N}(h_iI_i) + \sum_{X_i > 0} a_i \leq B?$$

---

**Cost components**

Ordering cost

Holding cost

---

**Problem parameters**

Planning horizon length: $N$

Period demands: $d_i$
The unlimited capacity deterministic production/inventory control problem can be stated as follows:

**INSTANCE** Number $N \in \mathbb{Z}^+$ of periods, for each period $i$, $1 \leq i \leq N$, a demand $d_i \in \mathbb{Z}_0^+$. A production set-up cost $a_i \in \mathbb{Z}^+$, and an inventory cost coefficient $h_i \in \mathbb{Z}^+$, an overall bound $B \in \mathbb{Z}^+$.

**QUESTION** Do there exist production amounts $X_i \in \mathbb{Z}_0^+$, and associated inventory levels $I_i = \sum_{t=1}^{i} (X_t - d_t)$, $1 \leq i \leq N$, such that

$$
\sum_{i=1}^{N} (h_i I_i) + \sum_{X_i > 0} a_i \leq B ?
$$

**Cost components**
- Ordering cost
- Holding cost

**Problem parameters**
- Planning horizon length: $N$
- Period demands: $d_i$
Unlimited capacity Deterministic Production Planning Problem [Wag – 58]

Cost components

Ordering cost $a$
Holding cost $h$

Problem parameters

Planning horizon length: $N$
Period demands: $d_i$
Unlimited capacity Deterministic Production Planning Problem [Wag – 58]

**Cost components**
- Ordering cost
- Holding cost

**Problem parameters**
- Planning horizon length: $N$
- Period demands: $d_i$
The capacitated deterministic production/inventory control problem can be stated as follows:

**INSTANCE** Number $N \in \mathbb{Z}^+$ of periods, for each period $i$, $1 \leq i \leq N$, a demand $d_i \in \mathbb{Z}_0^+$ and a production capacity $c_i \in \mathbb{Z}_0^+$. A production set-up cost $a_i \in \mathbb{Z}^+$, and an inventory cost coefficient $h_i \in \mathbb{Z}^+$, an overall bound $B \in \mathbb{Z}^+$.

**QUESTION** Do there exist production amounts $X_i \in \mathbb{Z}_0^+$ and associated inventory levels $I_i = \sum_{t=1}^{i} (X_t - d_t)$, $1 \leq i \leq N$, such that $X_i \leq c_i$, $1 \leq i \leq N$ and

\[
\sum_{i=1}^{N} (h_i I_i) + \sum_{X_i > 0} a_i \leq B ?
\]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1, c_1$</td>
<td>$d_i, c_i$</td>
<td>$d_N, c_N$</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>N</td>
</tr>
</tbody>
</table>

**Cost components**

- Ordering cost
- Holding cost

**Problem parameters**

- Planning horizon length: $N$
- Period demands: $d_i$
- Production capacity: $c_i$
Unlimited capacity Stochastic Production Planning

The unlimited capacity stochastic production/inventory control problem can be stated as follows:

**INSTANCE** Number $N \in \mathbb{Z}^+$ of periods, for each period $i$, $1 \leq i \leq N$, a random demand $d_i \in \mathbb{Z}_0^+$. A production set-up cost $a_i \in \mathbb{Z}^+$, an inventory cost coefficient $h_i \in \mathbb{Z}^+$, a shortage cost coefficient $p_i \in \mathbb{Z}^+$, and an overall bound $B \in \mathbb{Z}^+$.

**QUESTION** Do there exist production amounts $X_i \in \mathbb{Z}_0^+$ and associated inventory levels $I_i = \sum_{t=1}^{i}(X_t - d_t)$, $1 \leq i \leq N$, such that

$$
\sum_{i=1}^{N}(h_i I_i^+) + \sum_{i=1}^{N}(p_i I_i^-) + \sum_{X_i > 0} a_i \leq B?
$$

### Cost components

- Ordering cost
- Holding cost
- Shortage cost

### Problem parameters

- Planning horizon length: $N$
- Coefficient of variation: $c_v$
- Expected demands: $d_i$
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

Cost components
- Ordering cost
- Holding cost
- Shortage cost

Problem parameters
- Planning horizon length: \( N \)
- Coefficient of variation: \( c_v \)
- Expected demands: \( d_i \)

Pro: REDUCES NERVOUSNESS
Vs: COST SUBOPTIMAL
Unlimited capacity Stochastic Production Planning – \((S^n,s^n)\) Policy

Cost components

Ordering cost
Holding cost
Shortage cost

Problem parameters

Planning horizon length: \(N\)
Coefficient of variation: \(c_v\)
Expected demands: \(d_i\)
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- Replenishment cycle policy (R,S)
  - effective in damping *planning instability*, also known as *control nervousness*.
  - Silver [Sil – 98] points out that this policy is appealing in several cases:
    - Items ordered from the same supplier (joint replenishments)
    - Items with resource sharing
    - Workload prediction
    - ...

- Dynamic (R,S), heuristic approach [Boo – 88]
  - Considers a non-stationary demand over an N-period planning horizon
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

Cost components
- Ordering cost
- Holding cost
- Shortage cost

Problem parameters
- Planning horizon length: $N$
- Coefficient of variation: $c_v$
- Expected demands: $d_i$
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

Cost components

Ordering cost
Holding cost
Shortage cost

Problem parameters

Planning horizon length: \( N \)
Coefficient of variation: \( c_v \)
Expected demands: \( d_i \)
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

Cost components
- Ordering cost
- Holding cost
- Shortage cost

Problem parameters
- Planning horizon length: $N$
- Coefficient of variation: $c_v$
- Expected demands: $d_i$
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

Cost components
Ordering cost
Holding cost
Shortage cost

Problem parameters
Planning horizon length: \( N \)
Coefficient of variation: \( c_v \)
Expected demands: \( d_i \)
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

Problem parameters
- Planning horizon length: $N$
- Coefficient of variation: $c_v$
- Expected demands: $d_i$

Cost components
- Ordering cost
- Holding cost
- Shortage cost
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

Cost components

- Ordering cost
- Holding cost
- Shortage cost

Problem parameters

- Planning horizon length: \( N \)
- Coefficient of variation: \( c_v \)
- Expected demands: \( d_i \)
A deterministic equivalent [Bir – 97] CP formulation is [RTHP – 07]:

$$\min E\{TC\} = C$$  \hspace{1cm} (18)

subject to

$$\text{objConstraint}\left(C, \bar{I}_1, \ldots, \bar{I}_N, \delta_1, \ldots, \delta_N, d_1, \ldots, d_N, a, h, s\right)$$  \hspace{1cm} (19)

and for \(t = 1 \ldots N\)

$$\bar{I}_t + \bar{d}_t - \bar{I}_{t-1} \geq 0$$  \hspace{1cm} (20)

$$\bar{I}_t + \bar{d}_t - \bar{I}_{t-1} > 0 \Rightarrow \delta_t = 1$$  \hspace{1cm} (21)

$$\bar{I}_t \in \mathbb{Z}, \quad \delta_t \in \{0, 1\}$$  \hspace{1cm} (22)

The objective function (18) minimizes the expected total cost over the given planning horizon. $\text{objConstraint}(\cdot)$ dynamically computes buffer stocks and it assigns to $C$ the expected total cost related to a given assignment for replenishment decisions, depending on the demand distribution in each period and on the given combination for problem parameters $a, h, s$. 
Unlimited capacity Stochastic Production Planning
– Replenishment Cycle Policy

- Dedicated cost-based filtering techniques (see [Foc – 99]) can be developed
- In [Tar – 07] we already presented a similar filtering method under a service level constraint [Tar – 05, Tar – 04].
  - Dynamic programming relaxation [Tar – 96].
- This technique lets us solve instances with planning horizons up to 50 periods typically in less than a second for the service level case [Tar – 07]
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- **Cost-based filtering by relaxation**: the nodes in the shortest path are the optimal replenishment periods.

- (i) negative orders are scheduled (infeasible policy)
- (ii) no negative order arises (feasible and optimal policy)
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- **Cost-based filtering by relaxation:**
  - (a) \( i \) is not a replenishment period
Cost-based filtering by relaxation:

(a) \( i \) is not a replenishment period

We remove from the network every inbound (and outbound) arc to (from) node \( i \). By doing this we will look for a shortest path that is not passing through node \( i \).
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy

- Cost-based filtering by relaxation:
- (b) \( i \) is a replenishment period

1 \( \to \) \( i \) \( \to \) \( N+1 \)
Cost-based filtering by relaxation:

(b) \( i \) is a replenishment period

We remove from the network every arc \((h,j)\), where \( h<i<j \). By doing this we will look for a shortest path that is passing through node \( i \).
Cost-based filtering by relaxation:
(c) the expected closing inventory level at the end of period $i$ has been assigned to $I_i^*$

The expected total cost of cycles covering period $i$ is fixed
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy
Experimental results

<table>
<thead>
<tr>
<th>Test Set P1</th>
<th>Test Set P2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Table 1" /></td>
<td><img src="image" alt="Table 1" /></td>
</tr>
</tbody>
</table>

Test Set P3, P4

| ![Table 5](image) | ![Table 5](image) |
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy
Conclusions

- We presented a **cost based filtering technique** for a CP approach that finds optimal \((R^n, S^n)\) policies under nonstationary demands and a shortage cost scheme.

- Our computational experiments prove the effectiveness of our approach.
Unlimited capacity Stochastic Production Planning – Replenishment Cycle Policy References