Scheduling Internal Audit Activities: A Stochastic Combinatorial Optimization Problem

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Summary

- **Motivations**
- Problem Statement
- A MIP model
- Computational Results
- A CP model
- Computational Results
- Extensions
- Conclusions

Internal Control

- Increasing **competitive pressures** and **regulations** led firms to implement set of procedures to **control their operations**
  - **Internal Control**

- **Internal control** is often implemented for safeguarding assets and assure reliability of information flows
  - **Internal auditing** (Hughes, 1997) for accounting control(s)

Internal Control

- **Planning** "*plays a crucial role in ensuring the effectiveness and proper focus of the activities of an internal audit department*" (Boritz and Broca, 1986)

- **Allocating resources**
  - **audit time**
  - **staff**

  "*is the major focus of internal audit planning*" (Kanter, McEnroe and Kyes, 1990)

Internal Control

- Internal audit depts assess the audit planning problem using **two different approaches**:
  - Allocation of a fixed audit budget among multiple audit units
    - How much to spend on an auditable unit at fixed intervals of time rather than how often to audit units (Carey and Guest, 2000)
  - Timing of audit activities
    - How often to audit a unit, based on cost and benefits that change over time (Carey and Guest, 2000)

- We address the **internal audit planning problem** *using the second perspective*

- **Unlike previous models**, that determine optimal timing for **one audit unit**, our model determines the optimal timing of audit activities for **multiple audit units**

Audit Units

- Firms typically have **more than one auditable unit** to which audit resources have to be allocated
  - **Multiple lines of business**
  - **Several grouping criteria for businesses**

- **Auditable units** "*are the units upon which internal control procedures are applied to safeguard assets and assure the reliability of information flows*" (Ziegenfuss, 1995)
  - **Organizational units** (finance, accounting, department)
  - **Geographic regions** (branches, cities)
  - **Activities** (budgeting, purchasing, etc.)

Audit Risk

- Auditable units that have been identified should be **assessed in terms of risk factors**

- **Audit risk**: "*the possibility that an event or action may occur within an audit unit which would adversely affect the organization*" (Statement on Internal Auditing Standards, No 9)

- There are **several possible risk factors** to be considered
  - **Organizational size**
  - **Ethical climate**
  - **Competitive conditions**
  - **Etc.**

- Several techniques are available for **assessing risks**
  - **Davidson, 1976**
  - **Gray, 1983**
  - **Ziegenfuss, 1995**
  - **…**

Audit Loss

- **Audit risk** and **Audit Loss** are two related concepts
  – *"Failure to control audit risk results in losses"*
    - **Erroneous decisions**
    - **Record keeping**
    - **Financial losses**
  – *"Audit Loss is the effect of Audit Risk"* (Carey and Guest, 2000)

Audit Loss

- Compliance with controls within auditable units is assumed to **deteriorate naturally over time** unless appropriate action is taken at some point to **restore it to its proper levels**
  - such a deterioration manifests itself at the cost of accumulated
    - **Frauds**
    - **Errors**
    - **Inefficiencies**

- *A frequent internal auditing decreases the probability of transactions achieving a noncompliance state*
  - Losses reduction

- We will see that our formulation enables the audit depts to keep losses in the absence of auditing below a certain threshold

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Problem Statement

- We address the **audit scheduling problem** using the *Static-dynamic uncertainty strategy* developed by Bookbinder and Tan (1988) for inventory lot sizing problems
  - Tarim and Kingsman, 2004

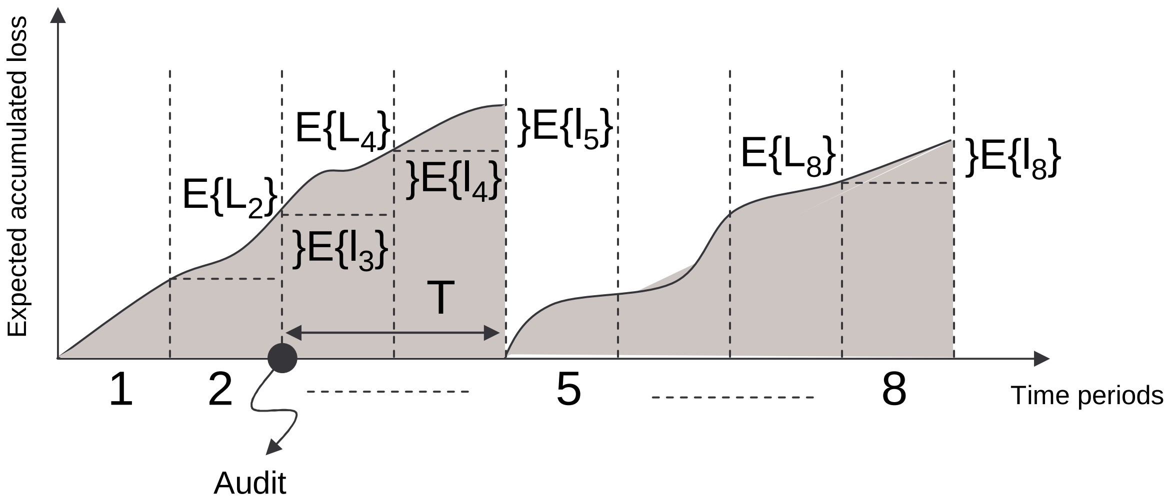
- Our objective is to find the **optimal audit schedule**
  - replenishment schedule in inventory

  by considering the **maximum loss level criterion**
  - service-level criterion in an inventory system

- The **fixed audit cost** and the **discounted expected total audit losses** are minimized by satisfying a **maximum loss level constraint**
  - we specify a minimum probability (α) that the loss will not exceed a predermined level in any given audit period

Problem Statement: inputs

- Fixed audit cost **a**
- A cut off, **L**, for losses and the respective probability α
- An audit time **T**
  - the time units required by the team to complete an audit
- Losses $\mathbf{l_{m,t}}$ that accrue in audit unit **m** during period **t**, a random variable with known probability density function $\mathbf{g_{m,t}(l_{m,t})}$. $\mathbf{E\{l_{m,t}\}}$ denotes the expected value of $\mathbf{l_{m,t}}$
  - we will assume for convenience and without loss of generality that **losses are normally distributed** with a constant coefficient of variation $\rho=\sigma_m/\mu_m$
  - the distribution may vary from period to period (**non-stationary**).
  - losses in different time periods are assumed to be **independent**



Problem Statement: model

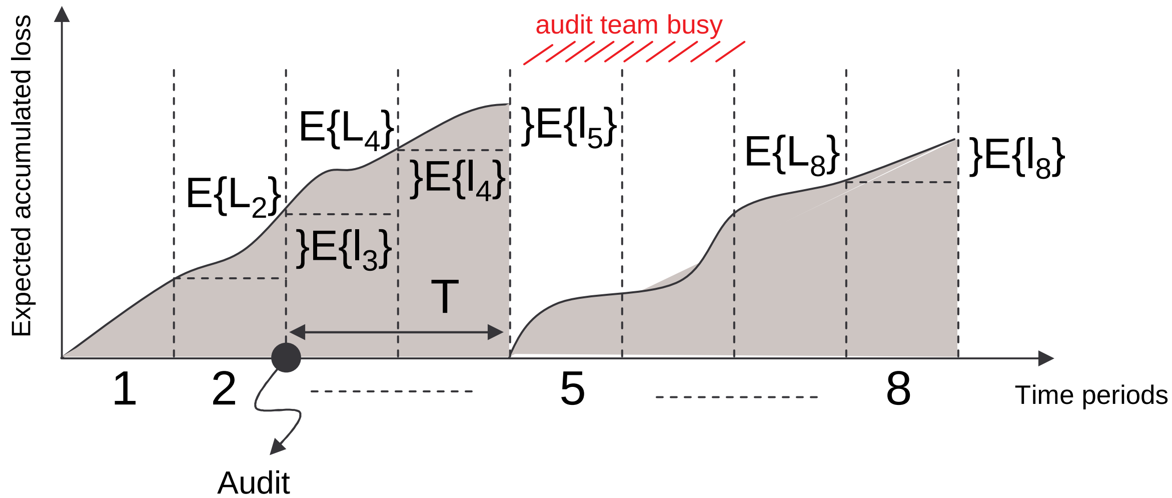
- The model balances the **discounted cost of losses accrued due to lack of audits** and the **cost of conducting audits**

- The **expected value criterion** is employed to minimize the **expected total losses and audit costs** over an *N* period planning horizon and M audit units

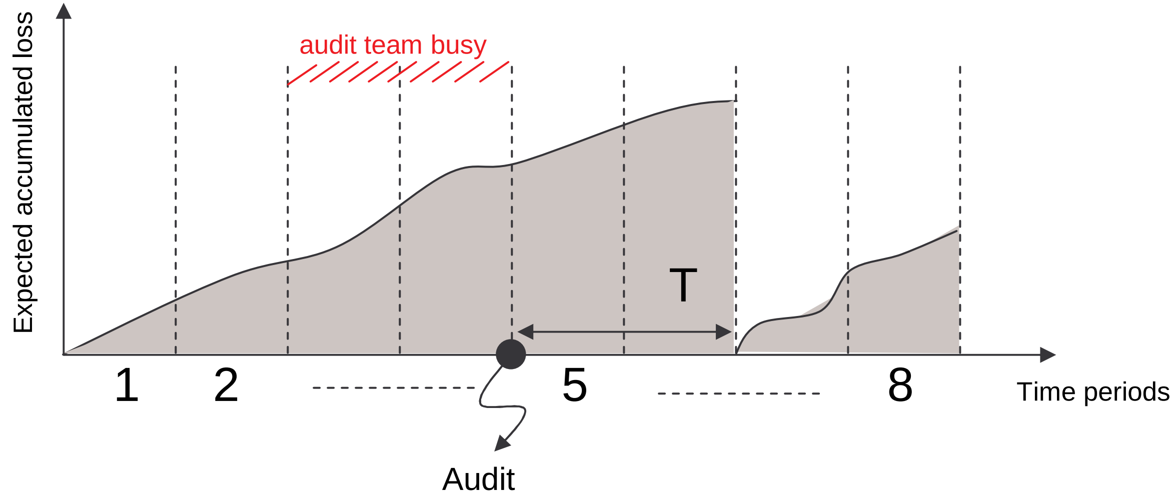
- The model provides the **optimum audit schedule for each audit unit**

Problem Statement: model

UNIT 1



UNIT 2



Problem Statement: model

- The **objective function** minimizes the expected total liabilities, E{TL}: expected loss + audit cost

$$\min E\{TL\} = \sum_{m=1}^{M} \int_{l_{m,1}} \int_{l_{m,2}} \dots \int_{l_{m,N}} \sum_{t=1}^{N} (aK_{m,t} + l_{m,t}) \\ g_{m,1}(l_{m,1}) g_{m,2}(l_{m,2}) \dots g_{m,N}(l_{m,N}) \mathrm{d}(l_{m,1}) d(l_{m,2}) \dots d(l_{m,N})$$

where

M : the total number of audit units,
N : the number of periods in the planning horizon,
a : the amount of cost incurred each time an audit is conducted,
$K_{m,t}$: a variable that takes the value of 1 if an internal audit is conducted in audit unit m in period t, otherwise 0.

Problem Statement: model

- **Audit cycle loss** can be defined as the losses that are expected to accrue between two consecutive audits.
- We also define

$L_{m,t}$: the loss level in audit unit m at the beginning of period t.

- We can modify the objective function presented in order to **incorporate a loss discount factor**, **h**, minimizing the sum of discounted period losses, $\mathbf{L_{m,t}}$, and audit cost

$$\min E\{TL\} = \sum_{m=1}^{M} \int_{l_{m,1}} \int_{l_{m,2}} \dots \int_{l_{m,N}} \sum_{t=1}^{N} (aK_{m,t} + h(L_{m,t} + l_{m,t}))$$
$$g_{m,1}(l_{m,1}) g_{m,2}(l_{m,2}) \dots g_{m,N}(l_{m,N}) \mathrm{d}(l_{m,1}) d(l_{m,2}) \dots d(l_{m,N})$$

  – This is more realistic and particularly suitable in all those cases where the **cost of money is an issue**:
    - Ex (1): *company accounts are wrong and there are capitals invested which in fact are not available → cost of borrowing money to cover current investments*
    - Ex (2): *company accounts are wrong and tax liabilities are overestimated → the capital tied in tax liabilities could be invested in a more profitable way*

Problem Statement: model

- The **initial amount of loss** can be set to any non-negative value
- Other **constraints in the model** are

$$L_{m,t+1} \geq L_{m,t} + l_{m,t}, \quad m = 1, ..., M, \quad t = 1, ..., T,$$

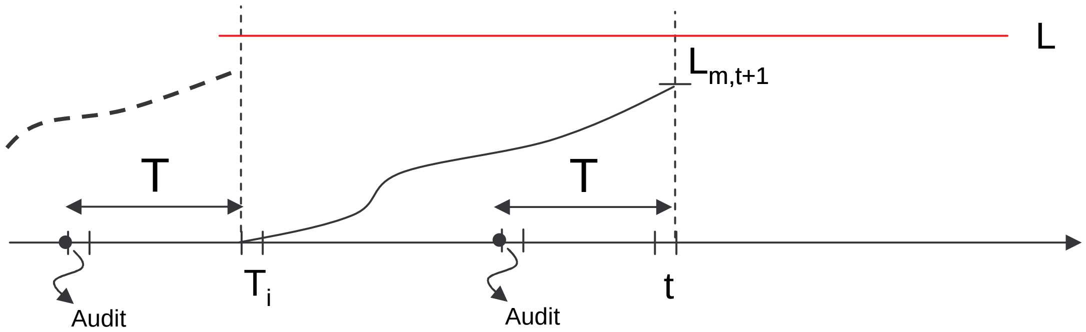
$$L_{m,t+1+T} \geq L_{m,t+T} + l_{m,t+T} - SK_{m,t+1}, \quad m = 1, ..., M, \quad t = 1, ..., N-T,$$

where **S** is some very large number and $\mathbf{l_{m,t}}$ is the amount of loss that accrues in audit unit **m** in period **t**

- Note that **K** is a binary variable defined as follows:
  - $\mathbf{K_{m,t+1}=1}$ → an audit is scheduled in period t+1 and will run for the next T periods
  - $\mathbf{K_{m,t+1}=0}$ → no audit is scheduled in period t+1

Problem Statement: model

- Let us assume that for audit unit **m**, the latest audit before period t has been completed by the beginning of period $\mathbf{T_i}$



the **max loss level constraint** can be espressed as

$$\Pr\{L_{m,t+1} \leq \bar{L}\} \geq \alpha, \quad t = 1, ..., N$$

where α is the *desired min probability that the loss level in any period will not exceed a subjectively predefined level* **L**

Problem Statement: model

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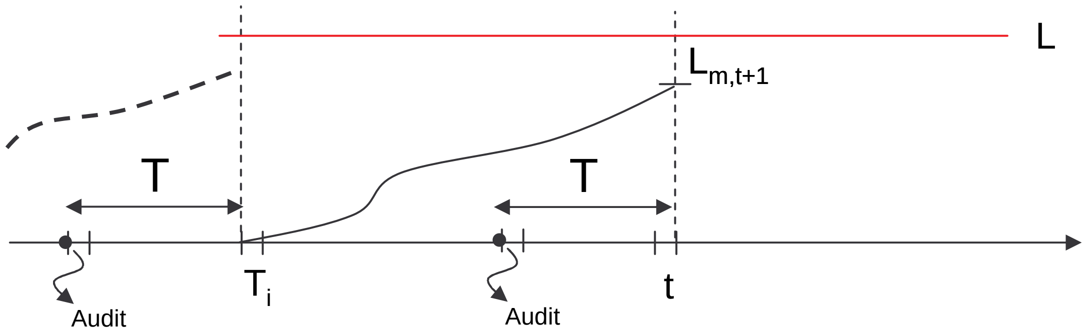

the **max loss level constraint** can be espressed as

$$\Pr\{\sum_{k=T_{m,i}}^{t} l_{m,k} \leq \bar{L}\} \geq \alpha, \quad t = 1, ..., N$$

where α is the *desired min probability that the loss level in any period will not exceed a subjectively predefined level* **L**

Problem Statement: model

- Since demands ($l_{m,i}$) in different periods are mutually independent it is easy to *compute the probability distribution function of the demand over periods* $\{T_i, \ldots, t\}$

- Let

$$G_{l_{m,T_{m,i}}+l_{m,T_{m,i}+1}+\ldots+l_{m,t}}(\bar{L})$$

  be the cumulative distribution function (CDF) of the demand in this time span
  - *this represents the probability that the demand in such a time span will be lower than a given value X*

- We assume this CDF to be **strictly increasing** and therefore $\mathbf{G^{-1}}$ to be defined

Problem Statement: model

- The former constraint can then be expressed as

$$\bar{L} \geq G^{-1}_{l_{m,T_{m,i}}+l_{m,T_{m,i+1}}+\ldots+l_{m,t}}(\alpha) \quad T_{m,i} \leq t < T_{m,i+1}, \quad i = 1, \ldots, r$$

  where **r** is the number of audits scheduled for unit **m**.

- Once a suitable probability distribution function for losses is chosen the right-hand side of the former equation can be **precomputed** and stored in a matrix for each possible audit cycle length

Problem Statement: model

- The right-hand side of Eq.

$$\bar{L} \geq G^{-1}_{l_{m,T_{m,i}}+l_{m,T_{m,i+1}}+\ldots+l_{m,t}}(\alpha) \quad T_{m,i} \leq t < T_{m,i+1}, \quad i = 1, \ldots, r$$

  can only be computed once an audit plan is fixed, but in order to fix an audit plan we need to know such values

- This circularity in the decision making process can be tackled by employing binary variables to select feasible audit cycles, that is cycles whose length does not violate the former probabilistic constraint on the max allowed loss level

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MIP Model

$$\min \quad \sum_{m=1}^{M} \left(\sum_{t=2}^{N} aK_{m,t} + \sum_{t=1}^{N} E\{l_{m,t}\} \right)$$

subject to $(m = 1, ..., M)$

$$E\{L_{m,1}\} = 0,$$
$$E\{L_{m,t+1}\} \geq E\{L_{m,t}\} + E\{l_{m,t}\}, \quad t = 1, ..., T$$
$$E\{L_{m,t+T}\} \geq E\{L_{m,t-1+T}\} + E\{l_{m,t-1+T}\} - SK_{m,t}, \quad t = 2, ..., N - T$$
$$\sum_{k=1}^{M} \sum_{h=2}^{\min(t+T,N)} K_{k,h} \leq 1, \quad t = 1, ..., N$$
$$\bar{L} \geq \sum_{j=1+T}^{t} G^{-1}_{l_{m,j}+l_{m,j+1}+...+l_{m,t}}(\alpha) \cdot P_{m,t,j}, \quad t = 1 + T, ..., N$$
$$\sum_{j=1}^{t} P_{m,t,j} = 1, \quad t = 1, ..., N$$
$$P_{m,t,j} \geq K_{m,j} - \sum_{k=j+1}^{t-T} K_{m,k}, \quad j = 1, ..., t - T$$
$$K_{m,1} = 1,$$
$$E\{L_{m,t}\} \geq 0, K_{m,t}, P_{m,t,j} \in \{0, 1\}, j = 1, ..., t$$

MIP Model

- The objective function **minimizes** the audit plan cost
  - fixed audit costs
  - cumulative loss

$$\min \quad \sum_{m=1}^{M} \left(\sum_{t=2}^{N} aK_{m,t} + \sum_{t=1}^{N} E\{l_{m,t}\} \right)$$

- Alternatively we can use the following one that employs discounted end of period losses

$$\min \quad \sum_{m=1}^{M} \left(\sum_{t=2}^{N} aK_{m,t} + h \sum_{t=1}^{N} (E\{l_{m,t}\} + E\{L_{m,t}\}) \right)$$

MIP Model

- Initial loss levels are set to 0 for convenience

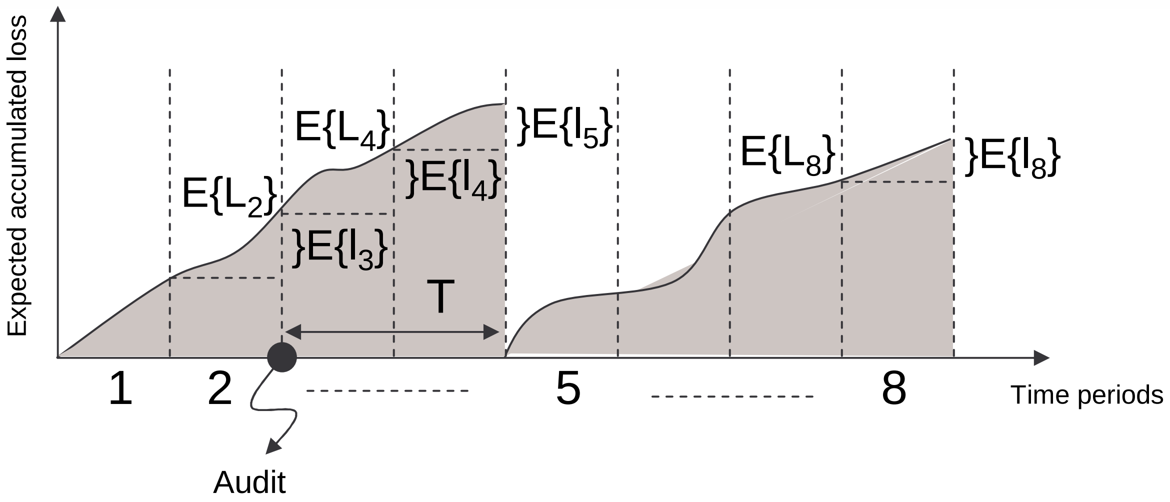
$$E\{L_{m,1}\} = 0,$$

- In the first **T** periods no audit can be completed (*the time required for performing an audit is, in fact, T*) therefore losses accumulate…

$$E\{L_{m,t+1}\} \geq E\{L_{m,t}\} + E\{l_{m,t}\}, \quad t = 1, ..., T$$

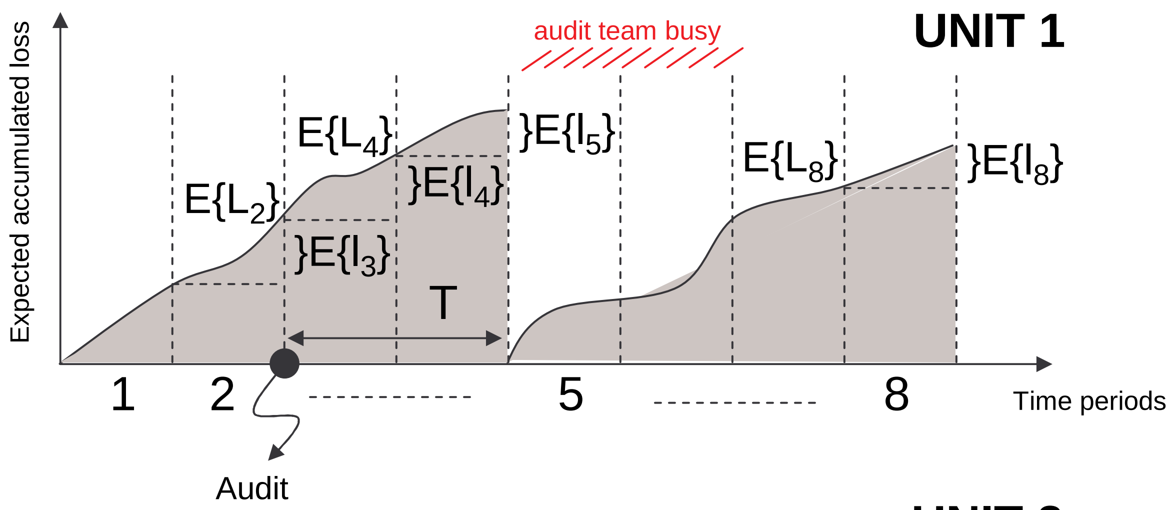
- afterwards, if no audit is scheduled in period **t**, losses accumulate from period **t+T-1** to period **t+T**, otherwise they are set to 0 in period **t+T**

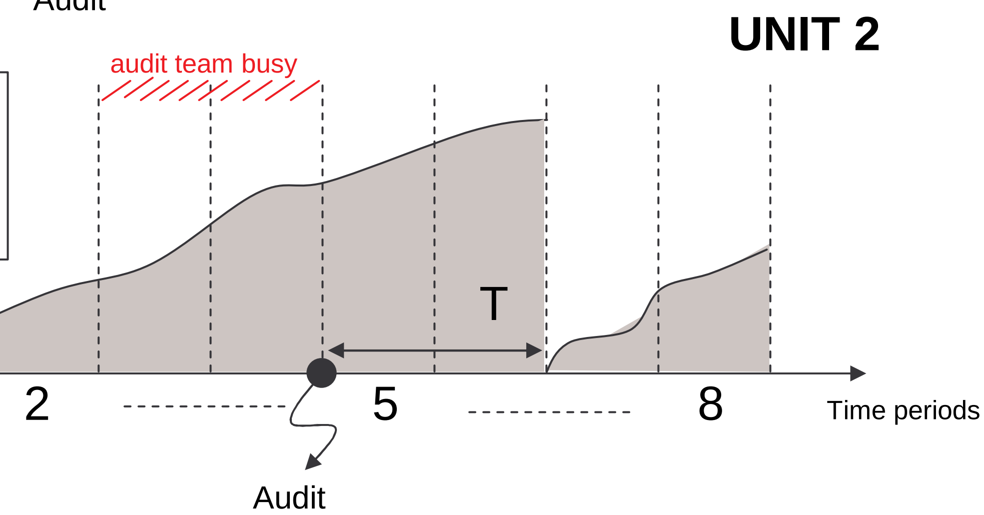
$$E\{L_{m,t+T}\} \geq E\{L_{m,t-1+T}\} + E\{l_{m,t-1+T}\} - SK_{m,t}, \quad t = 2, ..., N - T$$



MIP Model

- If an audit is scheduled for a unit in period **t** no other audit can be scheduled for any other unit in the next **T** periods:





$$\sum_{k=1}^{M} \sum_{h=2}^{\min(t+T,N)} K_{k,h} \le 1, \quad t = 1, ..., N$$

MIP Model

- The following set of equations identifies feasible audit cycles
  - *i.e. cycles for which the probability that accumulated losses will not exceed* **L** *is at least* α

$$\bar{L} \geq \sum_{j=1+T}^{t} G^{-1}_{l_{m,j}+l_{m,j+1}+\ldots+l_{m,t}}(\alpha)\cdot P_{m,t,j}, \quad t=1+T,\ldots,N$$

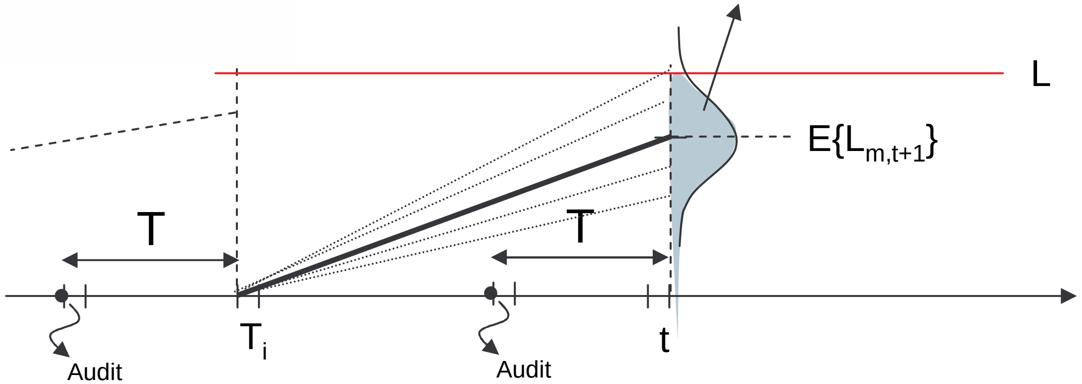
$$\sum_{j=1}^{t} P_{m,t,j} = 1, \quad t=1,\ldots,N$$

$$P_{m,t,j} \geq K_{m,j} - \sum_{k=j+1}^{t-T} K_{m,k}, \quad j=1,\ldots,t-T$$

$$K_{m,1} = 1,$$

We need a “padding” period at the beginning of the planning horizon

the probability associated to this area must be greater or equal to α



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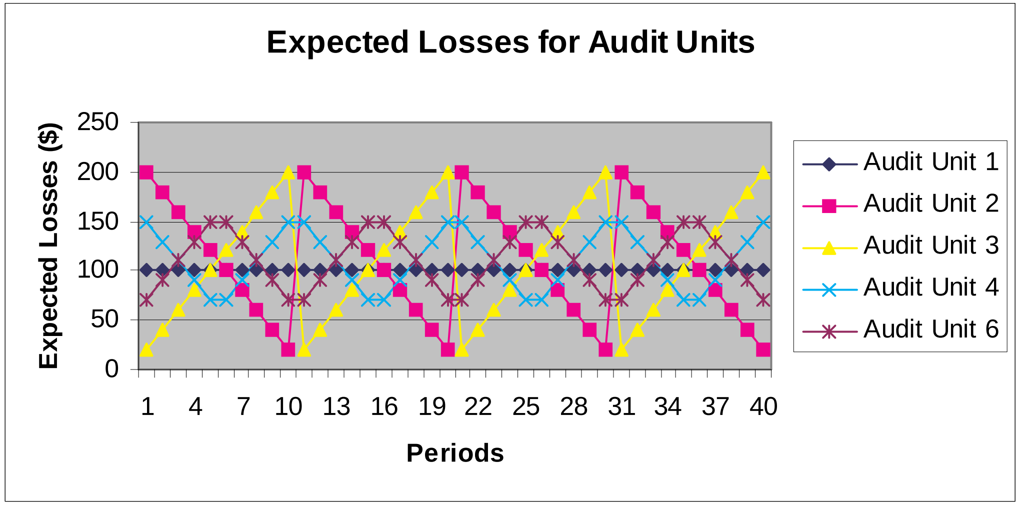
Computational Results

- We consider the following inputs:

- **5** audit units
- planning horizon length in {**20, 30, 40**}
- audit time (**T**) in {**1,…,6**}
- a thresold probability (α) of **95%**
- coefficient of variation (μ/σ) for the losses in each period in {**0.15, 0.3**}
- fixed audit cost (**a**) in {**500, 750, 1000**}
- threshold for accumulated losses (**L**) in {**1500, 2500, 3500**}
- expected losses:



MIP Performances (secs)

MIP										
μ/σ	1500			2500			3500			L
0,15	500	750	1000	500	750	1000	500	750	1000	a
T										N
1	2	1,6	0,6	2,4	1,1	0,6	1,9	1,1	0,6	10
2	86,5	138,3	131	460,4	209,5	245,64	322,2	426,9	375,13	20
3	77,63	45,1	58,2	36,8	33,5	35,4	80,4	52	74,7	20
4	139,2	43	102,4	124,1	397,1	320,7	666,3	612,3	756,5	30
5	prepr	prepr	prepr	1679,3	583,9	1980,8	248,9	164,1	208,6	30
6	prepr	prepr	prepr	4450,2	3600,7	3596,3	873	1152,4	2397	40

MIP										
μ/σ	1500			2500			3500			L
0,3	500	750	1000	500	750	1000	500	750	1000	a
T										N
1	1,8	1	0,6	2,8	1,2	0,7	1,7	1,2	0,5	10
2	116,2	107,7	197,1	498,3	307,3	347	253,7	328,8	316,7	20
3	19,1	37,1	81,5	41,1	35,1	41,9	81,4	53,1	47,2	20
4	70,6	93,1	75,45	923,27	273,5	401,1	808,7	743,1	730,3	30
5	prepr	prepr	prepr	397,39	600,35	869,4	229,1	311,1	221,1	30
6	prepr	prepr	prepr	3785,5	12549,2	8761,8	4660	5223,5	4457,7	40

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CP Model

$$\min \quad \sum_{m=1}^{M} \left(\sum_{t=2}^{N} aK_{m,t} + \sum_{t=1}^{N} E\{l_{m,t}\} \right)$$

subject to $(m = 1, ..., M)$

$$E\{L_{m,1}\} = 0,$$
$$E\{L_{m,t+1}\} \geq E\{L_{m,t}\} + E\{l_{m,t}\}, \quad t = 1, ..., T$$
$$K_{m,t} = 0 \leftrightarrow E\{L_{m,t+T}\} \geq E\{L_{m,t-1+T}\} + E\{l_{m,t-1+T}\}, \quad t = 2, ..., N - T$$
$$K_{m,t} = 1 \rightarrow E\{L_{m,t+T}\} = 0, \quad t = 2, ..., N - T$$
$$\sum_{k=1}^{M} \sum_{h=2}^{\min(t+T,N)} K_{k,h} \leq 1, \quad t = 1, ..., N$$
$$\Phi \left[m, t + T, \max \left(1, \max_{j=1,...,t} (j + T) \cdot K_{m,j} \right) \right] \geq 0, \quad t = 1, ..., N - T$$

where

$$\Phi[m, t, j] = \bar{L} - G^{-1}_{l_{m,j}+l_{m,j+1}+...+l_{m,t}}(\alpha), \quad t = 1, ..., N, \quad j = 1, ..., N$$

CP Model

- Initial loss levels are set to 0 for convenience

$$E\{L_{m,1}\} = 0,$$

- In the first **T** periods no audit can be completed (*the time required for performing an audit is, in fact,* **T**) therefore losses accumulate…

$$E\{L_{m,t+1}\} \geq E\{L_{m,t}\} + E\{l_{m,t}\}, \quad t = 1, ..., T$$

- afterwards, if no audit is scheduled in period **t**, losses accumulate from period **t+T-1** to period **t+T**,

$$K_{m,t} = 0 \leftrightarrow E\{L_{m,t+T}\} \geq E\{L_{m,t-1+T}\} + E\{l_{m,t-1+T}\}, \quad t = 2, ..., N-T$$

- otherwise they are set to 0 in period **t+T**

$$K_{m,t} = 1 \rightarrow E\{L_{m,t+T}\} = 0, \quad t = 2, ..., N-T$$

- *In MIP these two constraints where expressed as*

$$E\{L_{m,t+T}\} \geq E\{L_{m,t-1+T}\} + E\{l_{m,t-1+T}\} - SK_{m,t}, \quad t = 2, ..., N-T$$

CP Model

- The following set of equations identifies feasible audit cycles in the MIP model presented
  - *i.e. cycles for which the probability that accumulated losses will not exceed* **L** *is at least* α

$$\bar{L} \geq \sum_{j=1+T}^{t} G^{-1}_{l_{m,j}+l_{m,j+1}+\ldots+l_{m,t}}(\alpha) \cdot P_{m,t,j}, \quad t = 1+T, ..., N$$

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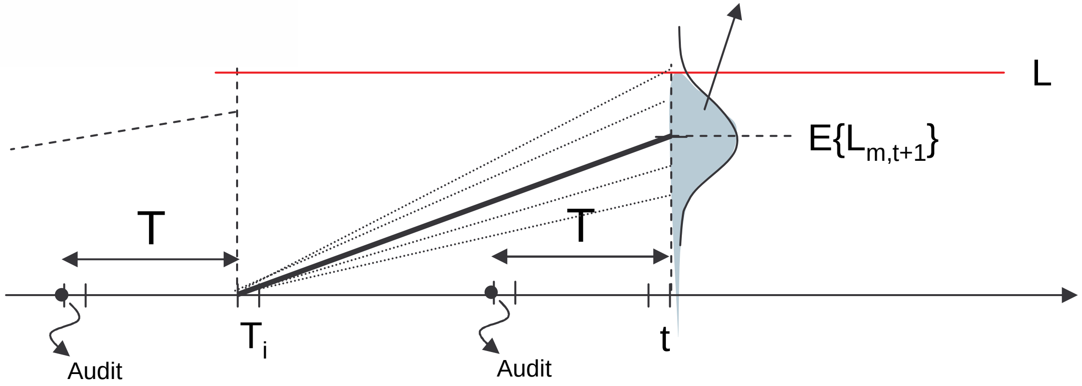
$$K_{m,1} = 1,$$

We need a “padding” period at the beginning of the planning horizon

the probability associated to this area must be greater or equal to α



CP Model

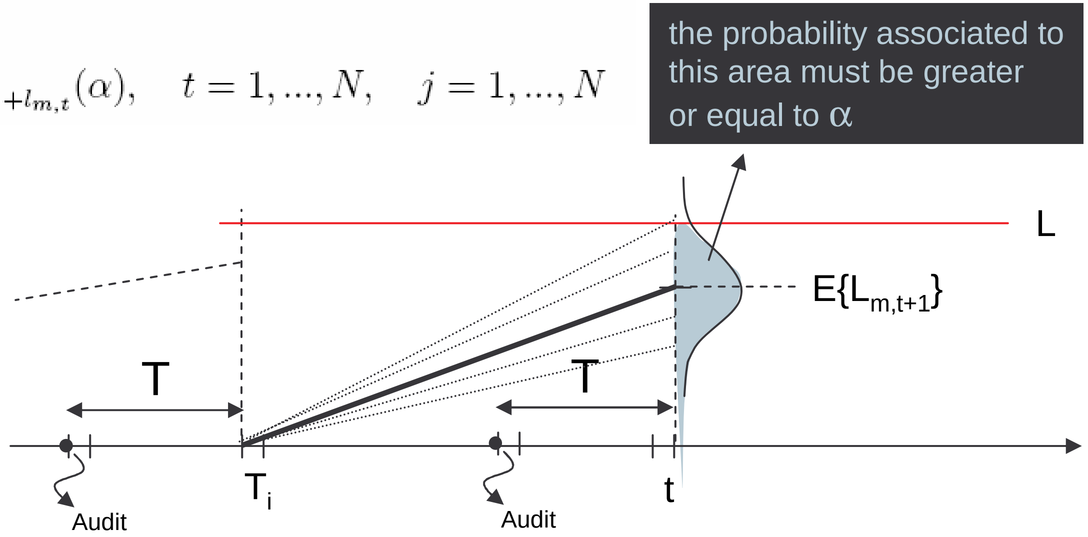
- Such a set of equations can be expressed in a more natural way using CP
  - *i.e. cycles for which the probability that accumulated losses will not exceed* **L** *is at least* α

$$\Phi\left[m, t+T, \max\left(1, \max_{j=1,\dots,t}(j+T)\cdot K_{m,j}\right)\right] \geq 0, \quad t = 1, \dots, N-T$$

where

$$\Phi[m,t,j] = \bar{L} - G^{-1}_{l_{m,j}+l_{m,j+1}+\dots+l_{m,t}}(\alpha), \quad t = 1, \dots, N, \quad j = 1, \dots, N$$

We do not need anymore a "padding" period at the beginning of the planning horizon



CP Mod...

Elements in the matrix can be efficiently indexed using the ELEMENT constraint (see Beldiceanu et al., "*Global Constraint Catalog")*

- Such a set o...
  natural way ...
  - *i.e. cycles for which ... umulated losses will not exceed ... least* α

$$\Phi\left[m, t+T, \max\left(1, \max_{j=1,\dots,t}(j+T)\cdot K_{m,j}\right)\right] \geq 0, \quad t=1,\dots,N-T$$

where

$$\Phi[m,t,j] = \bar{L} - G^{-1}_{l_{m,j}+l_{m,j+1}+\dots+l_{m,t}}(\alpha), \quad t=1,\dots,N, \quad j=1,\dots,N$$

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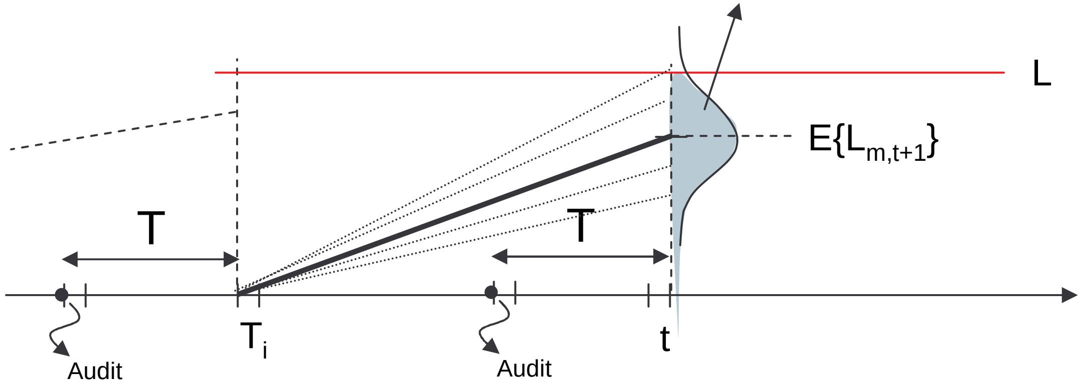

CP Model: considerations

- The CP approach presented, not only uses less variables and constraints, but it also results more natural in capturing the structure of the model

- **CP**
  - M(3N-2T)+N constraints
  - 2MN variables

- **MIP**
  - M((N^2)/2+3N-T+1)+N-1 constraints
  - 2MN+MN^2 variables

- Furthermore, as we shall see, CP is in practice more efficient than MIP both in proving **optimality** and **infeasibility**

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- A CP model
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CP Performaces (secs)

CP										
μ/σ	1500			2500			3500			L
0,15	500	750	1000	500	750	1000	500	750	1000	a
T										N
1	35,7	43,5	34,2	34,5	38,3	41,1	35,2	43,4	40,6	10
2	34,9	29,8	37,6	225,5	298,3	473,7	269,2	328,4	352,9	20
3	0,2	0,2	0,2	12,4	14,5	14,2	12,8	22,4	16,3	20
4	0,1	0,1	0,1	14,6	14,3	15,3	199,9	176,4	187,9	30
5	0,06	0,08	0,06	1,6	1,8	1,6	33,9	35,1	35,7	30
6	0,1	0,1	0,1	0,7	0,8	0,7	25,4	27,2	24,8	40

CP										
μ/σ	1500			2500			3500			L
0,3	500	750	1000	500	750	1000	500	750	1000	a
T										N
1	31,1	35,5	39,2	42,3	41,8	39,3	37,4	55,3	51,6	10
2	13,3	10,9	15,26	381,2	407,4	415,4	215,6	248,7	300,6	20
3	0,1	0,1	0,1	12,1	12,7	13,3	12,3	13,3	13,9	20
4	0,09	0,08	0,08	9	9,1	12,2	179,5	167	191,8	30
5	0,06	0,06	0,06	1,2	1,3	1,2	31,5	36,8	39,1	30
6	0,1	0,1	0,1	0,5	0,5	0,5	131,7	125,2	126,1	40

MIP Performances (secs)

MIP										
μ/σ	1500			2500			3500			L
0,15	500	750	1000	500	750	1000	500	750	1000	a
T										N
1	2	1,6	0,6	2,4	1,1	0,6	1,9	1,1	0,6	10
2	86,5	138,3	131	460,4	209,5	245,64	322,2	426,9	375,13	20
3	77,63	45,1	58,2	36,8	33,5	35,4	80,4	52	74,7	20
4	139,2	43	102,4	124,1	397,1	320,7	666,3	612,3	756,5	30
5	prepr	prepr	prepr	1679,3	583,9	1980,8	248,9	164,1	208,6	30
6	prepr	prepr	prepr	4450,2	3600,7	3596,3	873	1152,4	2397	40

MIP										
μ/σ	1500			2500			3500			L
0,3	500	750	1000	500	750	1000	500	750	1000	a
T										N
1	1,8	1	0,6	2,8	1,2	0,7	1,7	1,2	0,5	10
2	116,2	107,7	197,1	498,3	307,3	347	253,7	328,8	316,7	20
3	19,1	37,1	81,5	41,1	35,1	41,9	81,4	53,1	47,2	20
4	70,6	93,1	75,45	923,27	273,5	401,1	808,7	743,1	730,3	30
5	prepr	prepr	prepr	397,39	600,35	869,4	229,1	311,1	221,1	30
6	prepr	prepr	prepr	3785,5	12549,2	8761,8	4660	5223,5	4457,7	40

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Extensions for the CP Approach

- The CP model can be **extended** in several possible ways:
  - **Cost-based domain filtering**
    - Filippo Focacci, Andrea Lodi, Michela Milano: Cost-Based Domain Filtering. CP 1999: 189-203
  - **Ad-hoc value and variable selection heuristics**
    - Pascal Van Hentenryck, Laurent Perron, Jean-Francois Puget: Search and strategies in OPL. ACM Trans. Comput. Log. 1(2): 285-320
  - **Constraint-based local search**
    - Van Hentenryck, P. and Michel, L. 2005 *Constraint-Based Local Search*. The MIT Press.
  - **Scheduling constraints**
    - Beldiceanu, N., Carlsson, M., Demassey, S., and Petit, T. 2007. Global Constraint Catalogue: Past, Present and Future. *Constraints* 12, 1 (Mar. 2007), 21-62. DOI= http://dx.doi.org/10.1007/s10601-006-9010-8
  - *More…*

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Conclusions

- We proposed a **novel approach** for scheduling an audit team over several audit units under a maximum loss level chance constraint

- We developed a **MIP approach** for the problem

- We also proposed an **efficient and more natural way** of expressing the same problem **using CP**

- In our future work we plan to **incorporate in the CP** model the **extensions** that have been discussed

Questions

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Thanks!