Constraint Programming for Optimization under Uncertainty in Inventory Control

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- more accurate than other existing approaches in the literature
- more effective in terms of computational performance
- more effective in terms of expressiveness

Constraint Programming

Introduction

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- CP is related to **computer programming**
- Mathematical programming (MP) has nothing to do with computer programming
  - "Programming" historically refers to logistics plans, George Dantzig's first application
  - MP is purely **declarative**

Constraint Programming

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Sample CSP

- $V = \{x, y\}$
- $D(x) = \{1, 3, 4, 5\}$ $D(y) = \{4, 5, 8\}$
- $C = \{x + 3 = y\}$

A possible solution for the CSP is $x = 1$ and $y = 4$.

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Constraint Propagation

...is the **process** that repeatedly calls filtering algorithms until no new deduction can be made.

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during the search filtering algorithms are systematically applied when the domain of a variable is modified.

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Filtering strategies

- matching theory
- graph theory

- dynamic programming
- linear programming
- more...

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  - they are associated with powerful **filtering strategies**
  - new **custom** global constraints can be defined

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- θ_h is a threshold probability associated to chance-constraint h

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Global Chance-Constraints

Perhaps the most interesting aspect of SCP is that the concept of **global constraint** can be also adopted in a stochastic environment, thus leading to

- Global Chance-Constraints (Rossi et al., 2008)

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- A key question in inventory control is typically: “What is the optimal size for the orders issued”?
  - infrequent large orders?
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- This problem is known as **Lot sizing** problem and it constitutes a very active research area in combinatorial optimization

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- When the demand is assumed to be stochastic, the cost of insufficient capacity in the short run — that is the cost associated with shortages, or with averting them — assumes a great importance
- In stochastic lot-sizing the problem is typically to determine the “correct” quantity of buffer (or safety) stocks that must be kept to meet unexpected fluctuations of the demand

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The Newsvendor Problem

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- Therefore he must hedge against overage and underage costs in order to minimize the respective effects

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Inventory Control

The (R^n,S^n) policy

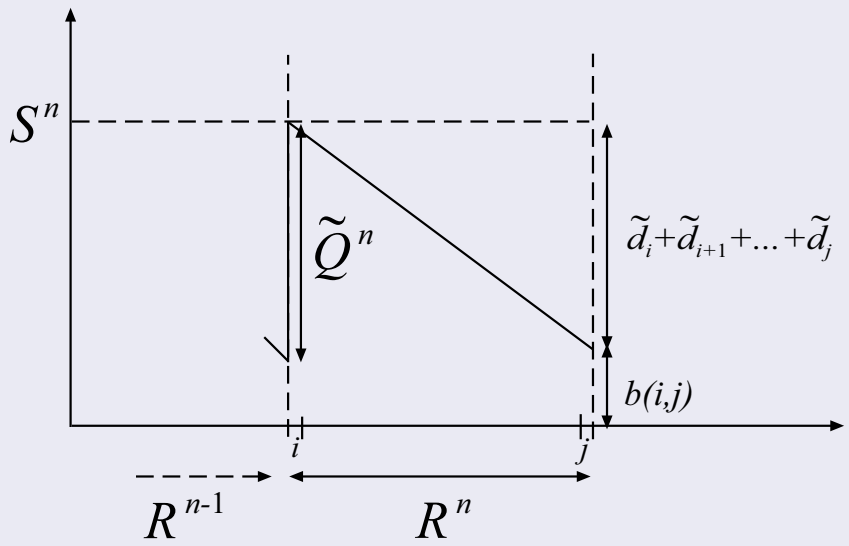

Figure: (R^n,S^n) policy. R^n denotes the set of periods covered by the nth replenishment cycle; S^n is the order-up-to-level for this cycle; $\tilde{Q}_n$ is the expected order quantity; $\tilde{d}_i + \tilde{d}_{i+1} + \ldots + \tilde{d}_j$ is the expected demand; $b(i,j)$ is the expected buffer stock

Global Chance-Constraints

There are three main contributions related to this novelty

- **Formal background** We have formally introduced *global chance-constraints*, defined as constraints that capture a relation among a non-fixed number of decision and random variables. These constraints not only are more expressive than the respective aggregation of simple chance-constraints, but they can be associated with more powerful filtering algorithms

Global Chance-Constraints

There are three main contributions related to this novelty

- **Application 1** We have applied *global chance-constraints* to compute optimal replenishment cycle policy parameters under non-stationary stochastic demand and service level constraints. *Global chance-constraints* allow the assumption on negative orders adopted in previous works [3, 5] to be relaxed and thus they let us compute the real optimal solution for the problem

Global Chance-Constraints

There are three main contributions related to this novelty

- **Application 2** We exploited *global chance-constraints* to represent multiple layers of uncertainty, demand uncertainty and delivery uncertainty, and to compute replenishment cycle policy parameters under non-stationary stochastic demand, service level constraints and stochastic delivery lag

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- a fixed delivery cost a is considered for each order
- a linear holding cost h is considered for each unit of product carried in stock from one period to the next
- a service level constraint — the probability that at the end of every period the net inventory will not be negative to be at least a given value α

Assumptions

The state-of-the-art formulation [3] operates under the assumption that negative orders are not allowed, so that if the actual stock exceeds the order-up-to-level for that review, this excess stock is carried forward and not returned to the supply source. However this event is assumed to be rare, therefore in the model it is ignored.

Assumptions

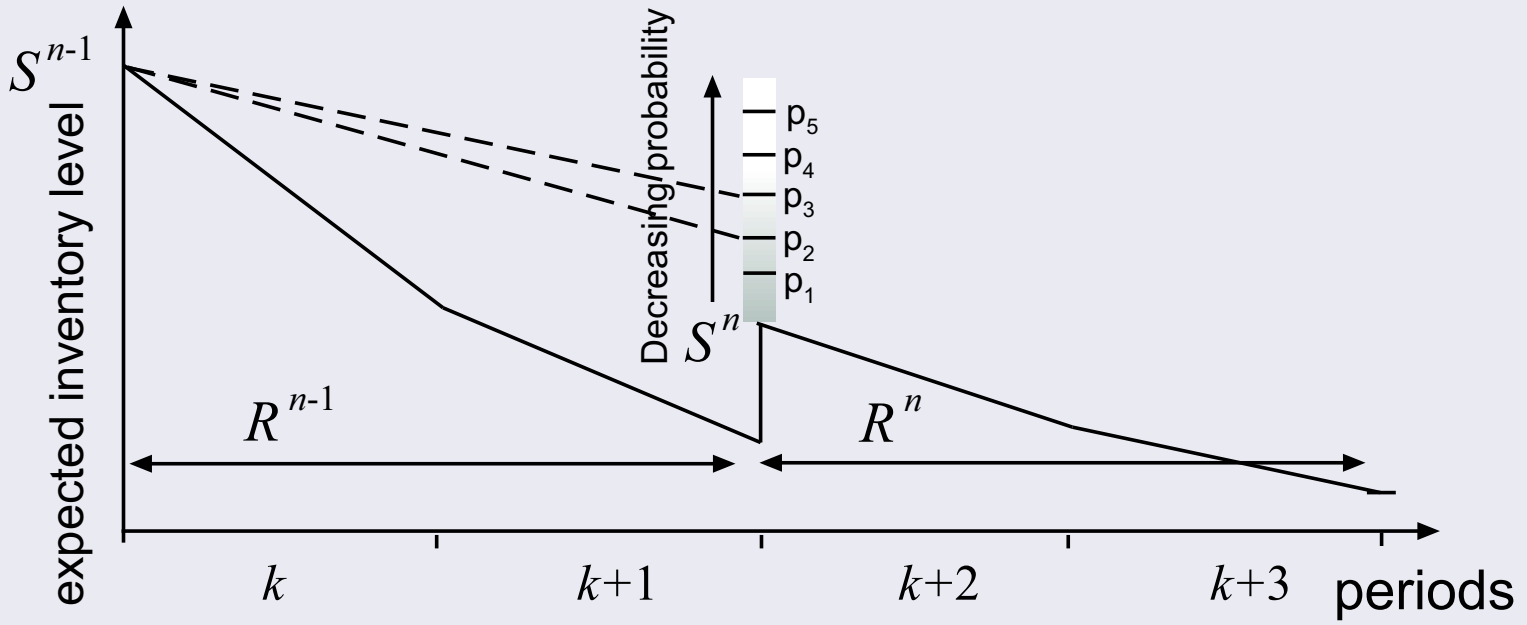

Figure: In Tarim & Kingsman [3] the event that actual stock exceeds the order-up-to-level S^n for a given review R^n is assumed to be rare. In other words, in their model observing a low demand during R^{n-1} has negligible probability. This implies that probabilities $p_1, p_2, \ldots, p_m$ are assumed to be low.

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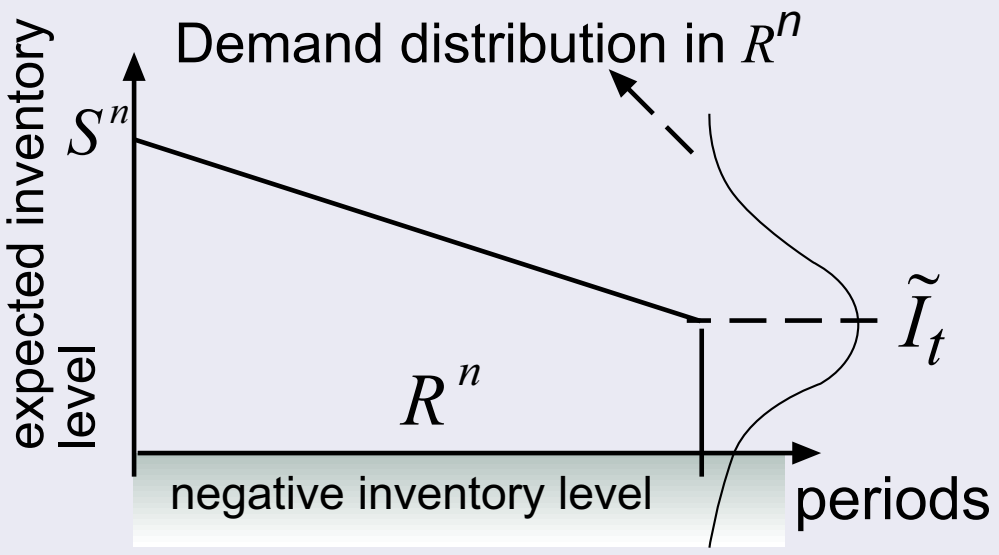

Figure: Negative inventory levels.

Stochastic CP model

The deterministic equivalent model that incorporates our constraint is

$$\min \; E\{TC\} = C \tag{1}$$

subject to

$$\mathit{serviceLevelRS}(C, a, h, \tilde{I}_{t\in\{1,\ldots,N\}}, \delta_{t\in\{1,\ldots,N\}}, d_{t\in\{1,\ldots,N\}}, \alpha) \tag{2}$$

and for $t = 1 \ldots N$,

$$\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} \geq 0 \tag{3}$$

$$\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} > 0 \Rightarrow \delta_t = 1 \tag{4}$$

$$\tilde{I}_t, C \in \mathbb{Z}^+ \cup \{0\}, \quad \delta_t \in \{0, 1\}. \tag{5}$$

Propagation (intuition...)

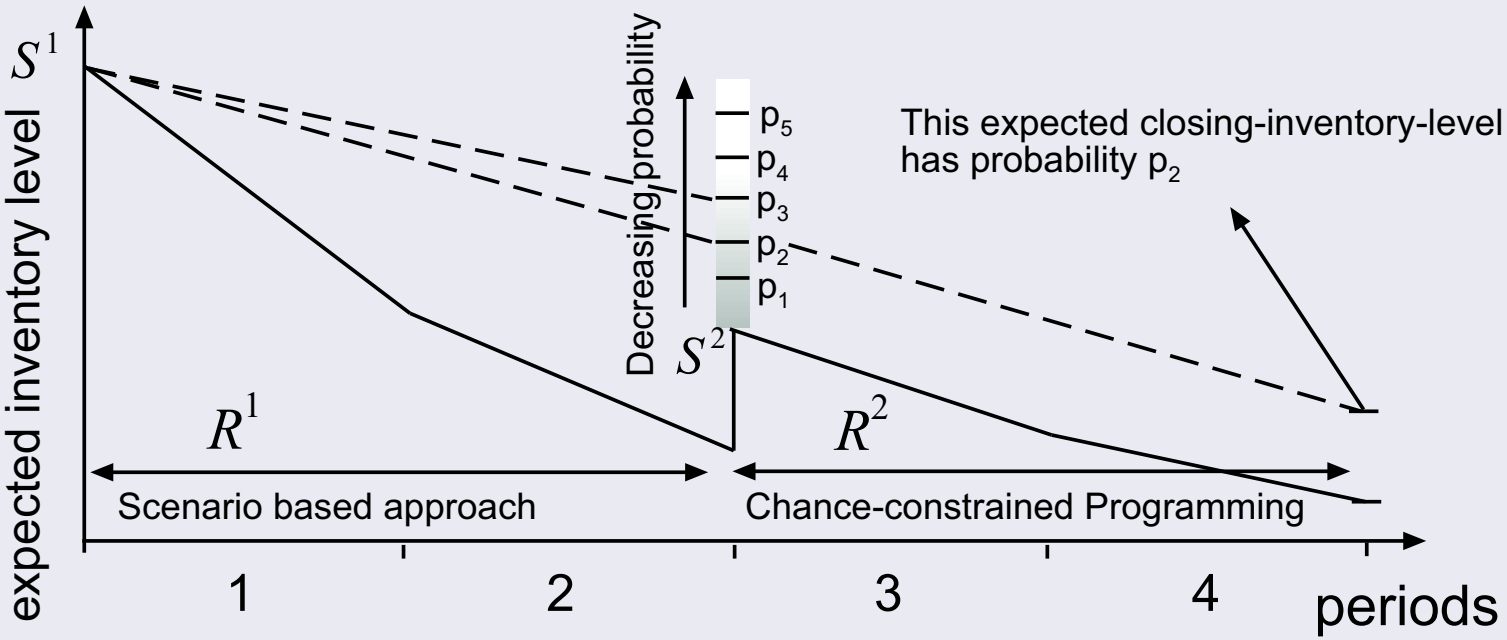

Figure: Two replenishment cycle case.

Improvement over the state-of-the-art model

				Total Cost						
	parameters			T&K				Exact		
	a	τ	α	$E\{TC\}$	$\widehat{E}\{TC\}$	gap(%)	sec	$E\{TC\}$	gap(%)	sec
1	1	0.25	0.95	324	370	12.4	1	358	3.35	469
2	100	0.25	0.95	773	814	5.04	1	799	1.88	254
3	200	0.25	0.95	1152	1189	3.11	1	1176	1.11	165
4	1	0.15	0.95	197	205	3.90	1	200	2.50	372
5	100	0.15	0.95	637	644	1.09	1	640	0.63	249
6	200	0.15	0.95	984	990	0.61	1	985	0.51	30
7	1	0.25	0.75	135	178	24.1	1	172	3.49	219
8	100	0.25	0.75	573	613	6.53	1	607	0.99	161
9	200	0.25	0.75	886	910	2.64	1	907	0.33	22
10	1	0.15	0.75	83	101	17.8	1	100	1.00	282
11	100	0.15	0.75	517	535	3.36	1	534	0.19	181
12	200	0.15	0.75	797	810	1.60	1	809	0.12	8

Table: Decreasing demand pattern.

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- We retain Tarim & Kingsman assumption on **negative order quantities**
- We charge holding cost on the inventory position and not on the inventory level
- We employed *Global Chance-Constraints* for computing feasible buffer stock levels

CP model

$$\min \quad E\{TC\} = \sum_{t=1}^{N} \left(a \cdot \delta_t + h \cdot \tilde{P}_t \right) + v \cdot \tilde{P}_N \tag{6}$$

subject to,

$$\tilde{P}_t + \tilde{d}_t - \tilde{P}_{t-1} > 0 \Rightarrow \delta_t = 1 \qquad t = 1, \ldots, N \tag{7}$$

$$\delta_t = 0 \Rightarrow \tilde{P}_t + \tilde{d}_t - \tilde{P}_{t-1} = 0 \qquad t = 1, \ldots, N \tag{8}$$

$$\tilde{P}_t + \tilde{d}_t - \tilde{P}_{t-1} \geq 0 \qquad t = 1, \ldots, N \tag{9}$$

$$\begin{aligned} &\mathit{serviceLevel}(\delta_1, \ldots, \delta_N, \\ &\qquad \tilde{P}_1, \ldots, \tilde{P}_N, \\ &\quad g_1(d_1), \ldots, g_N(d_N), \\ &\qquad f(\cdot), \alpha) \end{aligned} \tag{10}$$

$$\tilde{P}_t \geq 0, \quad \delta_t \in \{0, 1\} \qquad t = 1, \ldots, N. \tag{11}$$

Behind the scene...

$$\sum_{\omega_t \in \Omega_t} \Pr\{\omega_t\} \cdot G_S \left(R_{T_{p_\omega(t)}} + \sum_{\{i | i > p_\omega(t), (l_{T_i} | \omega_t) \leq t - T_i\}} (R_{T_i} - R_{T_{i-1}}) \right) \geq \alpha, \qquad (12)$$

$$t = L + 1, \ldots, N,$$

where $S = \sum_{k=T_{p_\omega(t)}}^{t} d_k - \sum_{\{i | i > p_\omega(t), (l_{T_i} | \omega_t) \leq t - T_i\}} (d_{T_{i-1}} + \ldots + d_{T_i - 1})$.

An example

We assume an initial null inventory level and a normally distributed demand with a coefficient of variation $\sigma_t/\tilde{d}_t = 0.3$ for each period $t \in \{1, \ldots, 5\}$. The expected values for the demand in each period are: $\{36, 28, 42, 33, 30\}$. The other parameters are $a = 1$, $h = 1$, $v = 0$, $\alpha = 0.95 (z_{\alpha=0.95} = 1.645)$. We consider for every period i in the planning horizon the following lead time probability mass function $f_i(t) = \{0.3, 0.2, 0.5\}$, which means that we receive an order placed in period i after $t \in \{0, \ldots, 2\}$ periods with the given probability (0 periods: 30%; 1 period: 20%; 2 periods: 50%). It is obvious that in this case we will always receive the order at most after 2 periods.

An example

S_1, $\Pr\{S_1\} = 0.15 = (0.3 + 0.2)0.3$; in this scenario at period t all the orders placed are received. That is the order placed in period $t-1$ is received immediately (probability 0.3), or after one period (probability 0.2), while the order placed in period t is received immediately (probability 0.3)

An example

S_2, $\Pr\{S_2\} = 0.35 = (0.3 + 0.2)(0.2 + 0.5)$; in this scenario at period t we don't receive the last order placed in period t. That is the order placed in period $t - 1$ is received immediately (probability 0.3), or after one period (probability 0.2), while the order placed in period t is not received immediately, therefore it is received after one period (probability 0.2), or after two periods (probability 0.5)

An example

S_3, $\Pr\{S_3\} = 0.35 = 0.5(0.2 + 0.5)$; in this scenario at period t we don't receive the last two orders placed in periods t and $t-1$. That is the order placed in period $t-1$ is received after two periods (probability 0.5), and the order placed in period t is not received immediately, therefore it is received after one period (probability 0.2), or after two periods (probability 0.5)

An example

S_4, $\Pr\{S_4\} = 0.15 = 0.5 \cdot 0.3$; in this scenario at period t we don't receive the order placed in period $t-1$ and we observe order-crossover. That is the order placed in period $t-1$ is received after two periods (probability 0.5), and the order placed in period t is received immediately (probability 0.3)

An example

Policy cost: 356					
Period (t)	1	2	3	4	5
$\tilde{d}_t$	36	28	42	33	30
R_t	125	124	129	87	55
δ_t	1	1	1	1	1
Shortage probability	–	–	5%	5%	5%

Table: Optimal solution.

An example

Let us consider period 3:

$$\Pr\{S_1\} \cdot G\left(\frac{129-42}{0.3\sqrt{42^2}}\right) + \Pr\{S_2\} \cdot G\left(\frac{124-(28+42)}{0.3\sqrt{28^2+42^2}}\right) + \Pr\{S_3\} \cdot G\left(\frac{125-(36+28+42)}{0.3\sqrt{36^2+28^2+42^2}}\right) + \Pr\{S_4\} \cdot G\left(\frac{125+(129-124)-(36+42)}{0.3\sqrt{36^2+42^2}}\right) = 94.60\% \cong 95\% \quad (13)$$

where $G(\cdot)$ is the standard normal distribution function. This means that the combined effect of order delivery delays in our policy, all possible scenarios taken into account, gives a no stock-out probability of about 95% for period 3.

Experiments

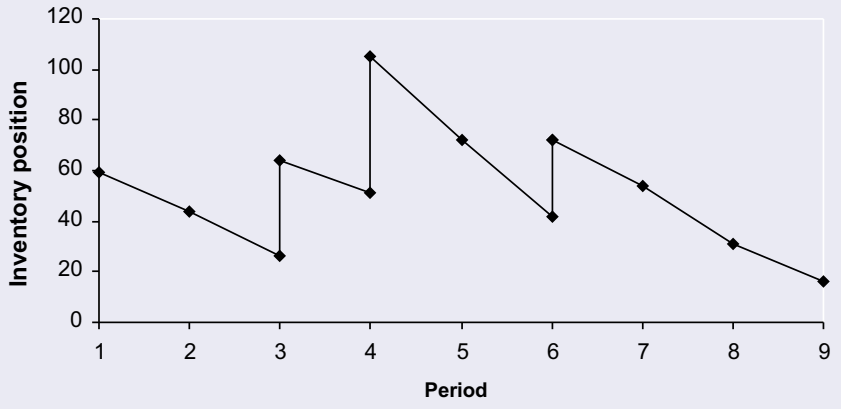

Figure: Optimal policy under deterministic one period lead time.

Experiments

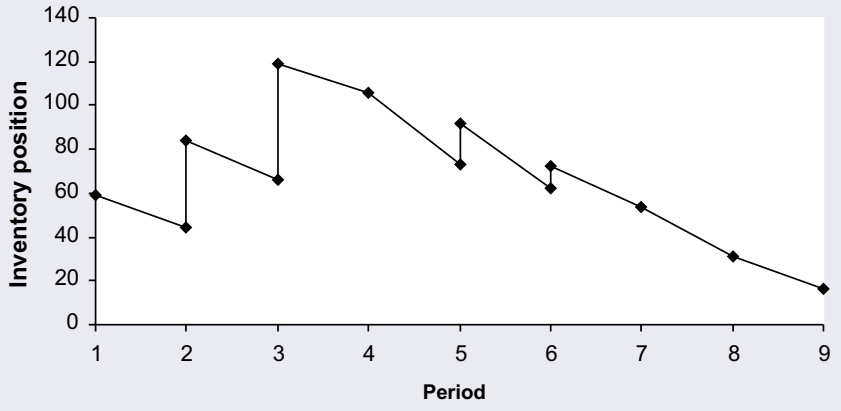

Figure: Optimal policy under deterministic two periods lead time.

Experiments

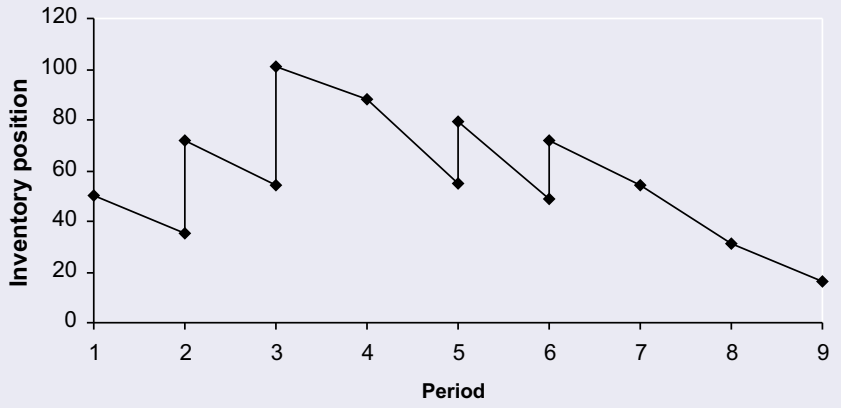

Figure: Optimal policy under stochastic lead time, $f_i(t) = \{0.2(0), 0.6(1), 0.2(2)\}$.

Experiments

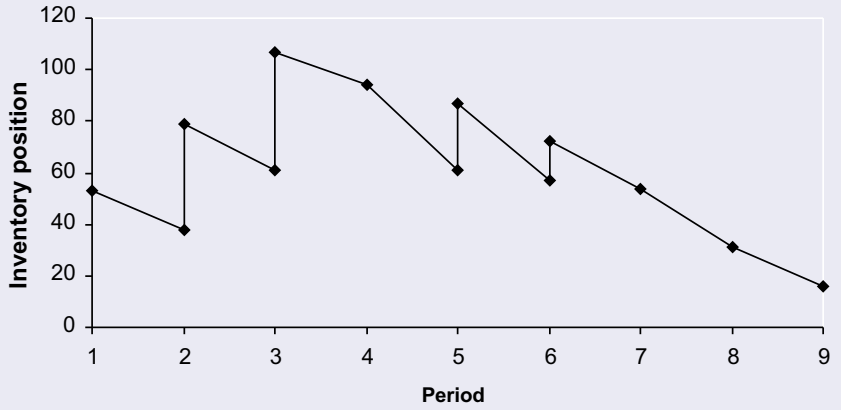

Figure: Optimal policy under stochastic lead time, $f_i(t) = \{0.5(0), 0.0(1), 0.5(2)\}$.

Optimization-Oriented Global Chance-Constraints

There are two main contributions related to this novelty

- **Formal background** We have formally introduced *optimization-oriented global chance-constraints*, defined as global chance-constraints that encapsulate suitable relaxations of the constraints considered. This relaxation, in contrast to conventional optimization-oriented global constraints, may involve stochastic variables

Optimization-Oriented Global Chance-Constraints

There are two main contributions related to this novelty

- **Application 3** By using *optimization-oriented global chance-constraints*, we have augmented the SCP model originally proposed by Tarim and Smith [5] for computing optimal replenishment cycle policy parameters under non-stationary stochastic demand and service level constraints. The augmented model produces run times that are orders-of-magnitude lower than those achieved by the state of the art approach in [5].

From Optimization-oriented global constraints...

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These pieces of information are exploited both for propagation purposes and for guiding the search.

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- At the **first stage** of relaxation, we are mainly involved with the stochastic variables and we exploit well known inequalities to replace stochastic variables in our stochastic programs with deterministic quantities and to yield a valid relaxation that is a deterministic problem.
- **A second stage of relaxation** may be needed to produce a problem that is computationally more tractable.

CP Model - Tarim & Smith

$$\min \; E\{TC\} = \sum_{t=1}^{N} \left(a\delta_t + h\tilde{I}_t \right) \tag{14}$$

subject to, for $t = 1 \ldots N$

$$\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} \geq 0 \tag{15}$$

$$\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} > 0 \Rightarrow \delta_t = 1 \tag{16}$$

$$\tilde{I}_t \geq b \left(\max_{j \in \{1,\ldots,t\}} j \cdot \delta_j, t \right) \tag{17}$$

$$\tilde{I}_t \in \mathbb{Z}^+ \cup \{0\}, \quad \delta_t \in \{0,1\}, \tag{18}$$

where $b(i,j)$ is defined by

$$b(i,j) = G^{-1}_{d_i + d_{i+1} + \ldots + d_j}(\alpha) - \sum_{k=i}^{j} \tilde{d}_k .$$

Extending Tarim & Smith's preprocessing

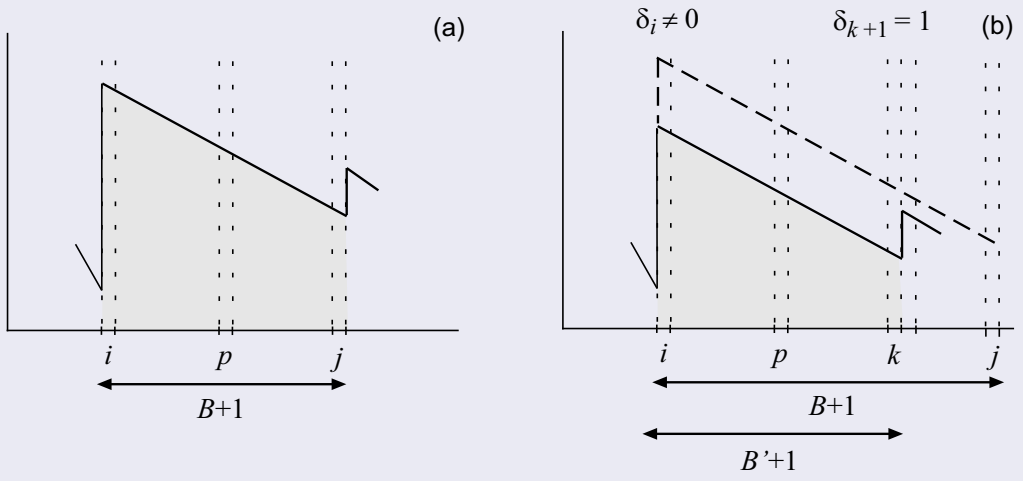
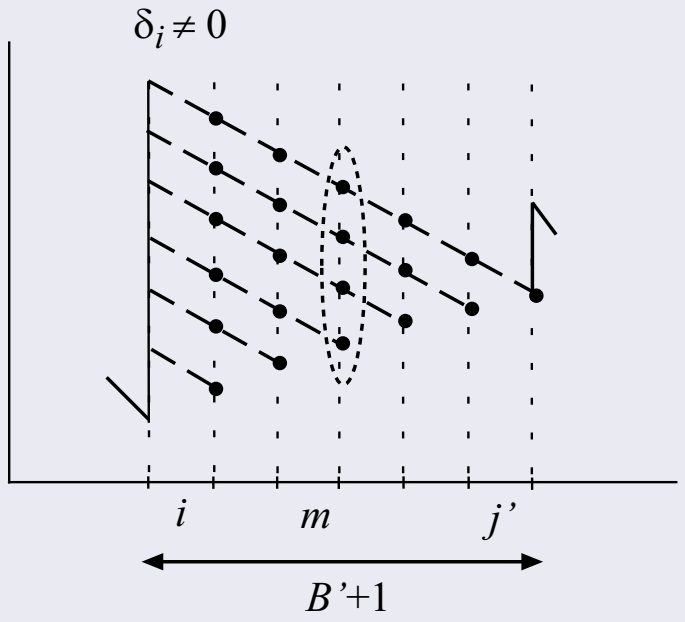


Figure: Bound tightening when a partial solution is given: (a) since it is not optimal to cover more than $B + 1$ periods with a single replenishment in i, the optimal policy lies in the gray area; (b) the bound B can be tightened to B' when an order is scheduled in period $k + 1$, $i \leq k < j$

Extending Tarim & Smith's preprocessing



A running example

We now present a running example where the planning horizon is $N = 24$ periods and the initial stock level is equal to zero. The demand is normally distributed in each period $t \in \{1, ..., N\}$ with a constant coefficient of variation $\sigma_t / \tilde{d}_t = 1/3$, where σ_t is the standard deviation of the demand in period t. The demand forecasts (mean value for each period) are listed in the following Table. The other parameters for the problem are: $a = 200$, $h = 1$, $\alpha = 0.95$.

A running example

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\tilde{d}_i$	73	0	128	116	92	180	28	164	28	161	37	57	181	62
i	15	16	17	18	19	20	21	22	23	24				
$\tilde{d}_i$	34	161	2	10	40	192	17	190	163	32				

Table: Demand forecasts

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14
δ_i	1	0	1	1	0	1	0	1	0	1	1	0	1	1
$\tilde{l}_i$	40	40	70	173	81	128	100	119	91	88	94	37	99	73
i	15	16	17	18	19	20	21	22	23	24				
δ_i	0	1	1	0	0	1	0	1	1	0				
$\tilde{l}_i$	39	88	86	76	36	123	106	104	123	91				

Table: Optimal solution

A running example

i	$Dom(\tilde{l}_i)$	i	$Dom(\tilde{l}_i)$
1	{40}	13	{99, 167}
2	{0, 40, 198}	14	{34, 37, 73, 105}
3	{70, 211}	15	{19, 39}
4	{64, 95, 173}	16	{88, 90, 100, 143}
5	{50, 81}	17	{1, 16, 73, 86, 88, 98, 141, 350}
6	{99, 128}	18	{5, 6, 63, 76, 78, 88, 131, 340}
7	{15, 71, 100}	19	{22, 23, 36, 38, 91, 300}
8	{90, 119}	20	{105, 108, 123}
9	{15, 62, 91}	21	{9, 88, 106}
10	{88, 128}	22	{104}
11	{20, 51, 91, 94}	23	{89, 123}
12	{31, 37}	24	{18, 57, 91}

Table: Reduced domains after applying our filtering method when no partial solution is given. The reduction achieved is equivalent to the one provided by pre-processing method I in [5]. Underlined figures are closing inventory levels of the optimal policy

A running example

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14
δ_i	1	0	1	–	0	1	0	1	0	–	–	0	1	–
i	15	16	17	18	19	20	21	22	23	24				
δ_i	0	1	1	0	0	1	0	1	–	0				

Table: Partial solution. A "–" means that the variable has not been assigned yet

A running example

i	$Dom(\tilde{l}_i)$	i	$Dom(\tilde{l}_i)$
1	{40}	13	{99, 167}
2	{40}	14	{34, 37, 73, 105}
3	{70, 211}	15	{39}
4	{64, 95, 173}	16	{88}
5	{81}	17	{1, 16, 73, 86}
6	{99, 128}	18	{6, 63, 76}
7	{100}	19	{23, 36}
8	{90, 119}	20	{105, 123}
9	{91}	21	{106}
10	{88, 128}	22	{104}
11	{20, 51, 91, 94}	23	{89, 123}
12	{37}	24	{91}

Table: Enforcing tighter upper bounds for optimal replenishment cycle lengths, underlined figures are closing inventory levels of the optimal policy

Generating good LB during the search

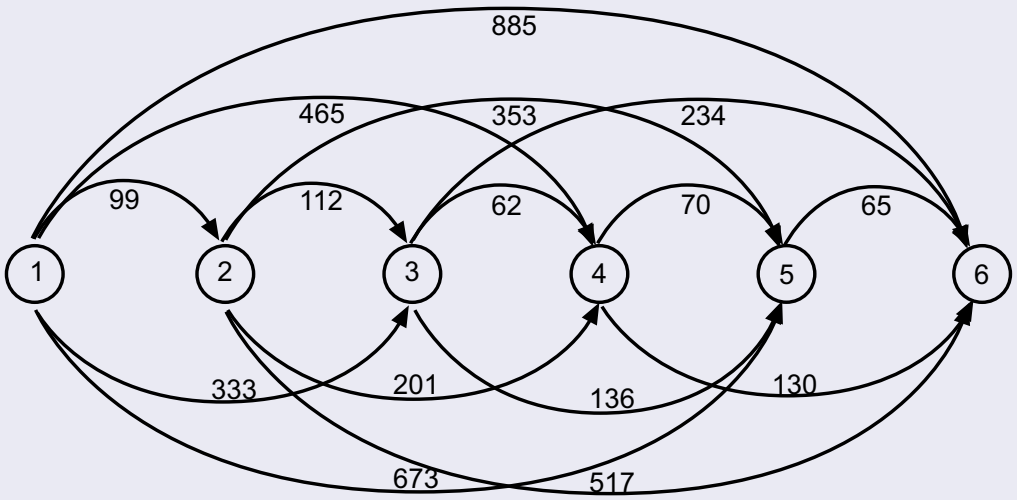


Figure: Shortest Path Relaxation

Generating good LB during the search

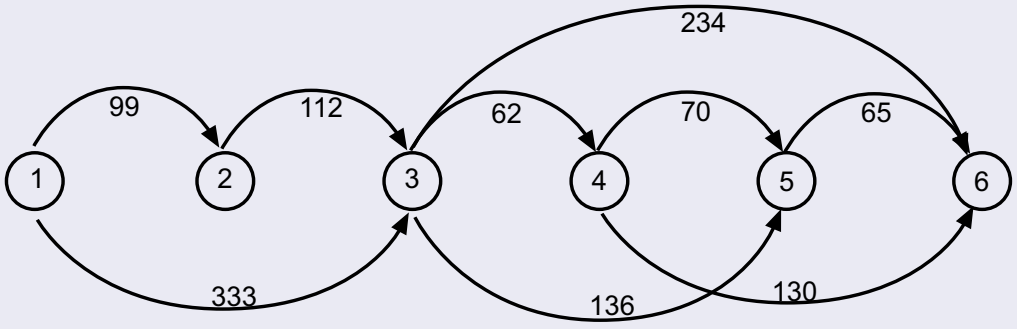


Figure: $\delta_3 = 1$

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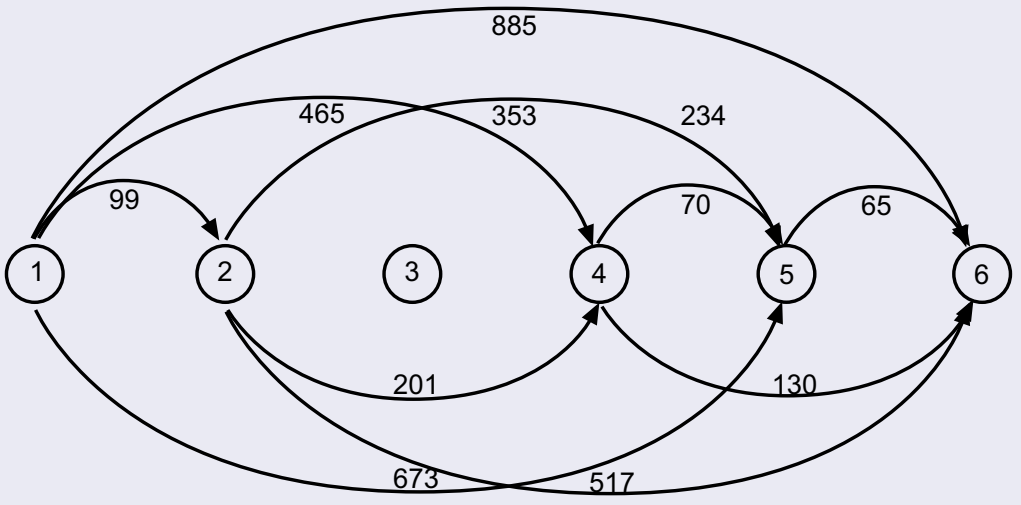

Figure: $\delta_3 = 0$

Experiments

		No Filt.		Method I		Method II		Method III		Combined	
α	a	Nod	Sec	Nod	Sec	Nod	Sec	Nod	Sec	Nod	Sec
0.95	40	127	1.85	96	1.64	96	1.43	120	1.30	70	1.12
	80	2994	30	1449	16	2586	23	82	1.02	63	0.97
	160	–	–	–	–	–	–	133	1.81	108	1.65
	320	–	–	–	–	–	–	4	0.09	4	0.09
0.99	40	261	3.27	198	4.24	202	2.52	253	2.84	165	2.57
	80	1234	11	611	7.54	1138	10.7	317	2.66	221	2.61
	160	–	–	–	–	–	–	168	2.15	84	1.31
	320	–	–	–	–	–	–	1	0.09	1	0.10

Table: Filtering methods compared in terms of explored nodes (“Nod”) and run time in seconds (“Sec”). Symbol “–” means that an optimal solution has not be found within the given limit of 60 secs

A Global Perspective

We have employed both *global chance-constraints* and *optimization-oriented global chance-constraints* to obtain the state of the art approach for computing replenishment cycle policy parameters under non-stationary stochastic demand and a penalty cost scheme

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- **Application 4** We have applied *global chance-constraints* to model the non-linear cost function that is only approximated by the approach in [4], which employs a piecewise linear approximation for modeling period holding and back-ordering costs. In addition to this we have applied *optimization-oriented global chance-constraints* to the same model in order to perform cost-based reasoning and thus improve the efficiency of the search process

Problem definition

We consider the class of production/inventory control problems that refers to the single location, single product case under non-stationary stochastic demand.

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- a fixed delivery cost a is considered for each order
- a linear holding cost h is considered for each unit of product carried in stock from one period to the next
- a linear shortage cost s for each unit of demand that is backordered

Tarim & Kingsman's model

$$\min\ E\{TC\} =$$

$$\int_{d_1}\int_{d_2}\dots\int_{d_N}\sum_{t=1}^{N}\left(a\delta_t + vX_t + hI_t^+ + sI_t^-\right) g_1(d_1)\dots g_N(d_N)\mathrm{d}(d_1)\dots\mathrm{d}(d_N) \tag{19}$$

subject to

$$X_t > 0 \Rightarrow \delta_t = 1 \tag{20}$$

$$I_t = \sum_{i=1}^{t}(X_i - d_i) \tag{21}$$

$$I_t^+ = \max(0, I_t) \tag{22}$$

$$I_t^- = -\min(0, I_t) \tag{23}$$

$$X_t, I_t^+, I_t^- \in \mathbb{Z}^+ \cup \{0\}, \quad I_t \in \mathbb{Z}, \quad \delta_t \in \{0,1\} \tag{24}$$

Cost analysis

Consider a review schedule which has m reviews over the N period planning horizon with orders arriving at $\{T_1, T_2, \ldots, T_m\}$, $T_j > T_{j-1}$.

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The expected cost function is the summation of m intervals, T_i to T_{i+1} for $i = 1, \ldots, m$, defining $D_{t_1,t_2} = \sum_{j=t_1}^{t_2} d_j$:

$$\min \; E\{TC\} = \sum_{i=1}^{m} \left(a\delta_{T_i} + \sum_{t=T_i}^{T_{i+1}-1} E\{C_{T_i,t}\} \right) + vI_N + v \int_{D_{1,N}} D_{1,N} \times g(D_{1,N}) \mathrm{d}(D_{1,N}), \tag{25}$$

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The term $v \int_{D_{1,N}} D_{1,N} \times g(D_{1,N}) \mathrm{d}(D_{1,N})$ is constant and can therefore be ignored in the optimization model

Cost analysis

$E\{C_{T_i,t}\}$ of Eq. (25) is defined as:

$$\int_{-\infty}^{S_{T_i}} h\left(S_{T_i} - D_{T_i,t}\right) g(D_{T_i,t})\mathrm{d}(D_{T_i,t}) - \int_{S_{T_i}}^{\infty} s\left(S_{T_i} - D_{T_i,t}\right) g(D_{T_i,t})\mathrm{d}(D_{T_i,t}). \quad (26)$$

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As stated in (Tarim and Kingsman [4]), $E\{C_{T_i,t}\}$ is the expected cost function of a single-period inventory problem where the single-period demand is $D_{T_i,t}$.

Cost analysis

Let $S^* - \mu = z_\beta \sigma$ be the safety stock.

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$$\begin{aligned} E\{TC(S^*)\} = h \cdot E\{S^* - D\}^+ + s \cdot E\{D - S^*\}^+ = \\ h \cdot (S^* - \mu) + (h+s)E\{D - S^*\}^+ = \\ h z_\beta \sigma + (h+s)\sigma E\{Z - z_\beta\}^+ = \\ h z_\beta \sigma + (h+s)\sigma[\phi(z_\beta) - (1-\beta)z_\beta] = \\ (h+s)\sigma\phi(z_\beta) \end{aligned} \tag{27}$$

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expression

$$h z_\alpha \sigma + (h+s)\sigma[\phi(z_\alpha) - (1-\alpha)z_\alpha] \tag{28}$$

can be used to compute the expected total cost for any given level S such that $\alpha = \Phi\left(\frac{S-\mu}{\sigma}\right)$.

Cost analysis

$E\{TC(S)\}$ as a function of the opening inventory level S.

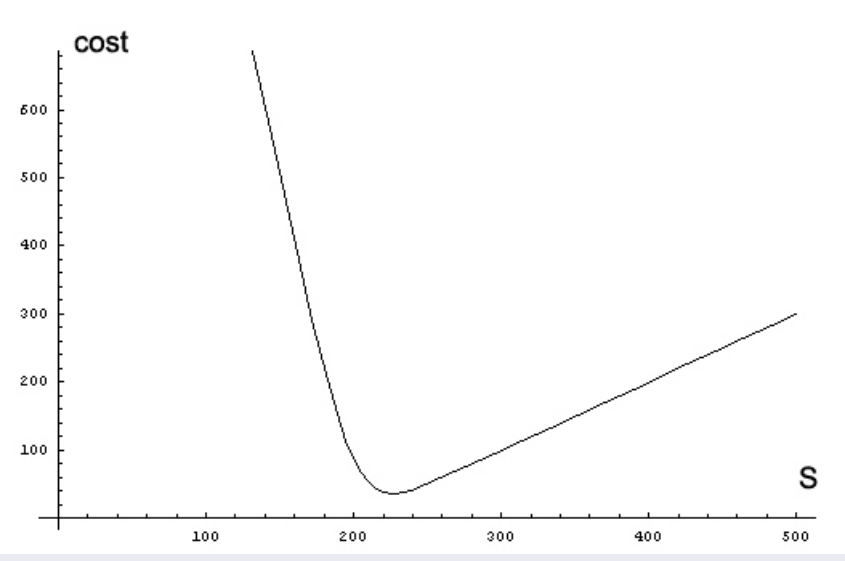

Figure: Single-period holding and shortage cost as a function of the opening inventory level S. The demand is normally distributed with mean 200 and standard deviation 20. Holding cost is 1, shortage cost is 10.

Cost analysis

The former equation can be extended in the following way to compute the cost for the replenishment cycle $R(i,j)$ as a function of the opening inventory level S

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The former equation can be extended in the following way to compute the cost for the replenishment cycle $R(i,j)$ as a function of the opening inventory level S

$$\sum_{k=i}^{j} \left(h z_{\alpha(i,k)} \sigma_{i,k} + (h+s)\sigma_{i,k}[\phi(z_{\alpha(i,k)}) - (1-\alpha(i,k))z_{\alpha(i,k)}] \right) \tag{29}$$

where $G_{i,k}(S) = \alpha(i,k)$ and $z_{\alpha(i,k)} = \Phi^{-1}(\alpha(i,k))$. Therefore we have $j-i+1$ cost components: the holding and shortage cost at the end of period $i, i+1, \ldots, j$.

Cost analysis

We plot this cost for a particular instance as a function of the opening inventory level S.

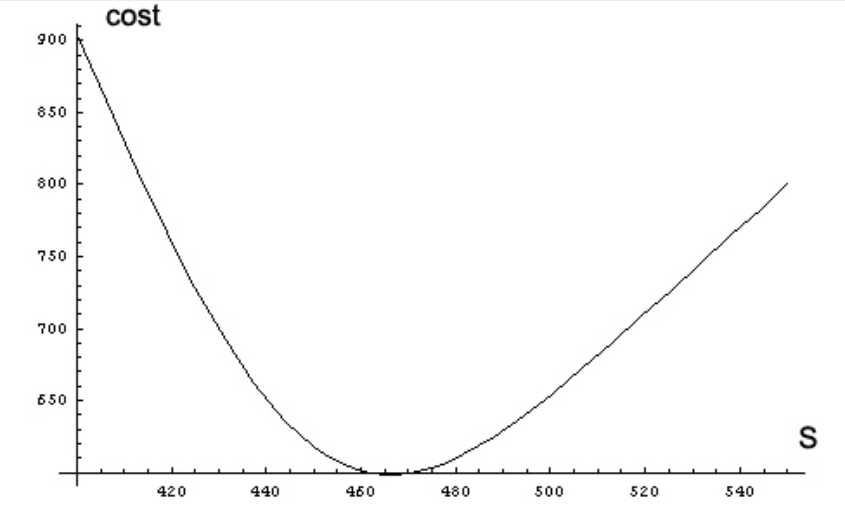

Figure: Three periods holding and shortage cost as a function of the opening inventory level S. The demand is normally distributed in each period with mean respectively 150, 100, 200, the coefficient of variation is 0.1. Holding cost is 1, shortage cost is 10.

Bounds

- Upper bound for opening inventory levels

Bounds

- Upper bound for opening inventory levels
  - ignore unit cost

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- Upper bound for opening inventory levels
  - ignore unit cost
  - ignore holding cost for intermediate periods

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Bounds

- Upper bound for opening inventory levels
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- Lower bound for expected closing inventory levels
  - optimize the convex cost of each possible replenishment cycle independently of the others
  - the minimum value obtained is the lower bound

CP model

$$\min \; E\{TC\} = C \tag{30}$$

subject to

$$\text{objConstraint}\left(C, \tilde{I}_1, \ldots, \tilde{I}_N, \delta_1, \ldots, \delta_N, d_1, \ldots, d_N, a, h, s\right) \tag{31}$$

and for $t = 1 \ldots N$

$$\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} \geq 0 \tag{32}$$

$$\tilde{I}_t + \tilde{d}_t - \tilde{I}_{t-1} > 0 \Rightarrow \delta_t = 1 \tag{33}$$

$$\tilde{I}_t \in \mathbf{Z}, \quad \delta_t \in \{0, 1\} \tag{34}$$

objConstraint

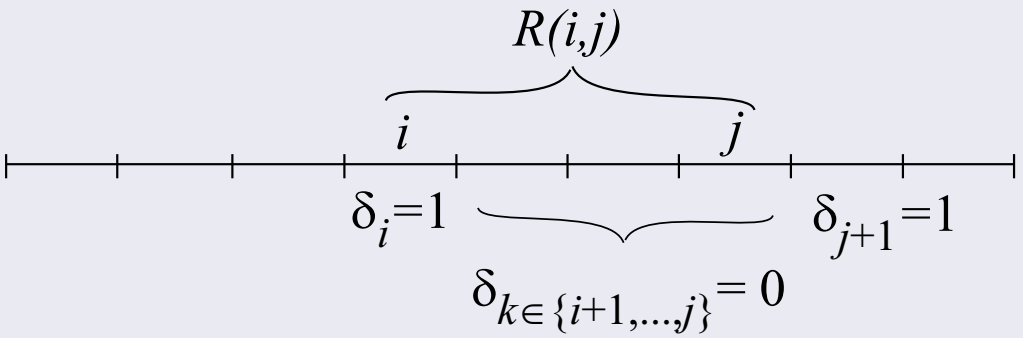

Figure: A replenishment cycle $R(i,j)$ is identified by the current partial assignment for δ_i variables.

objConstraint

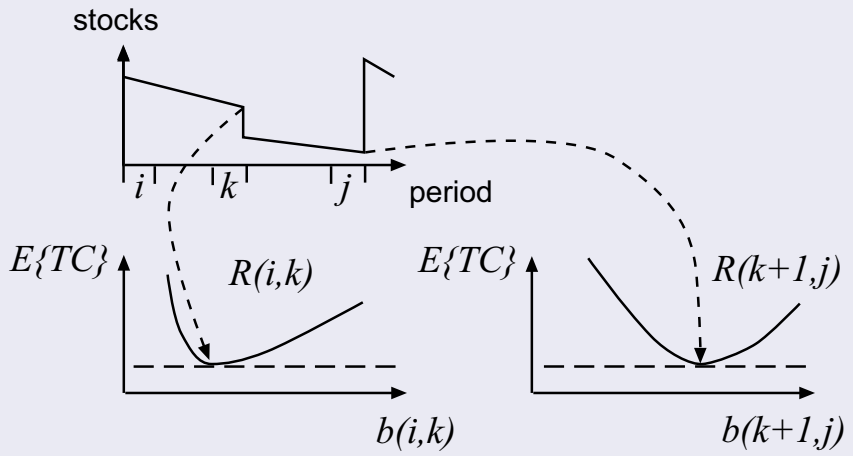

Figure: The expected total cost of both replenishment cycles is minimized, but the inventory conservation constraint is violated between $R(i,k)$ and $R(k+1,j)$

objConstraint

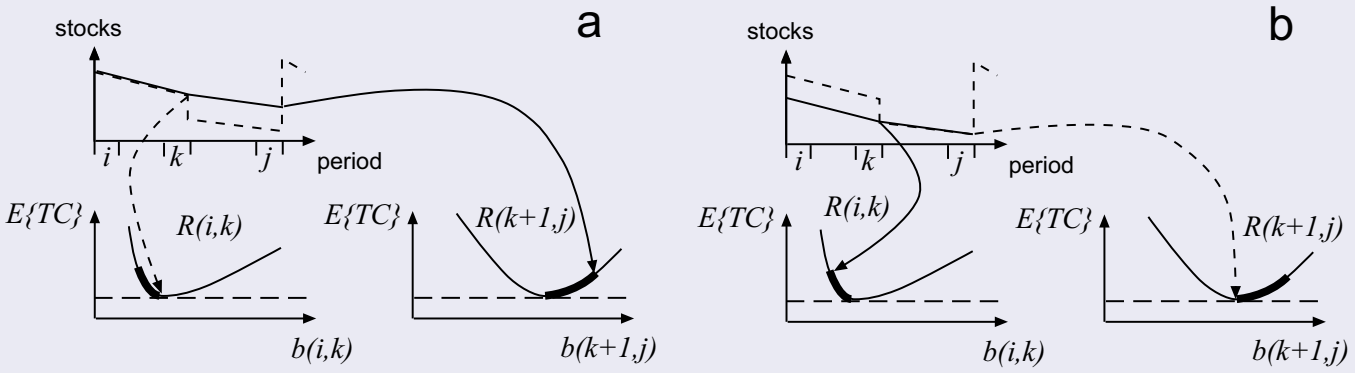

Figure: Feasible limit situations when negative order quantity scenarios arise

objConstraint

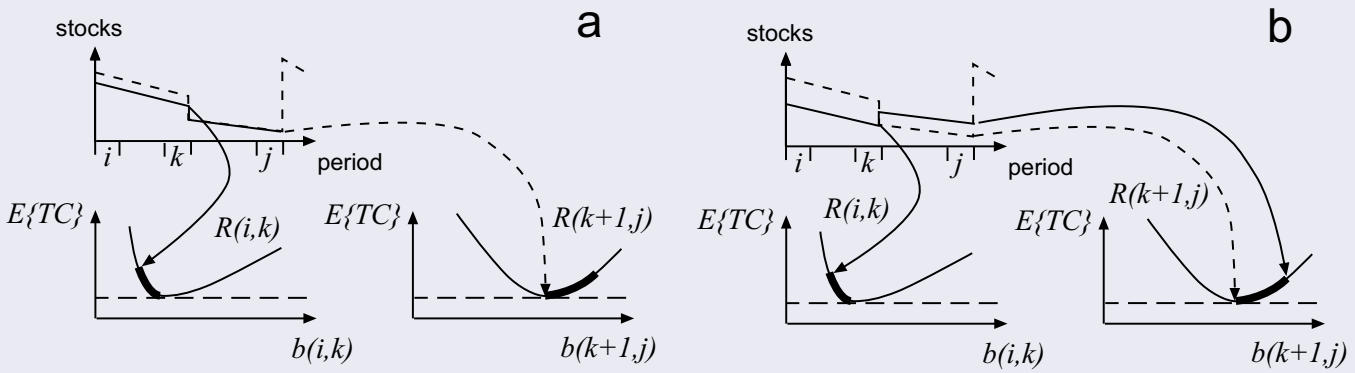

Figure: Infeasible (a) and suboptimal (b) plans realized when the opening inventory level of the second cycle doesn't equate the expected closing inventory level of the first cycle

objConstraint

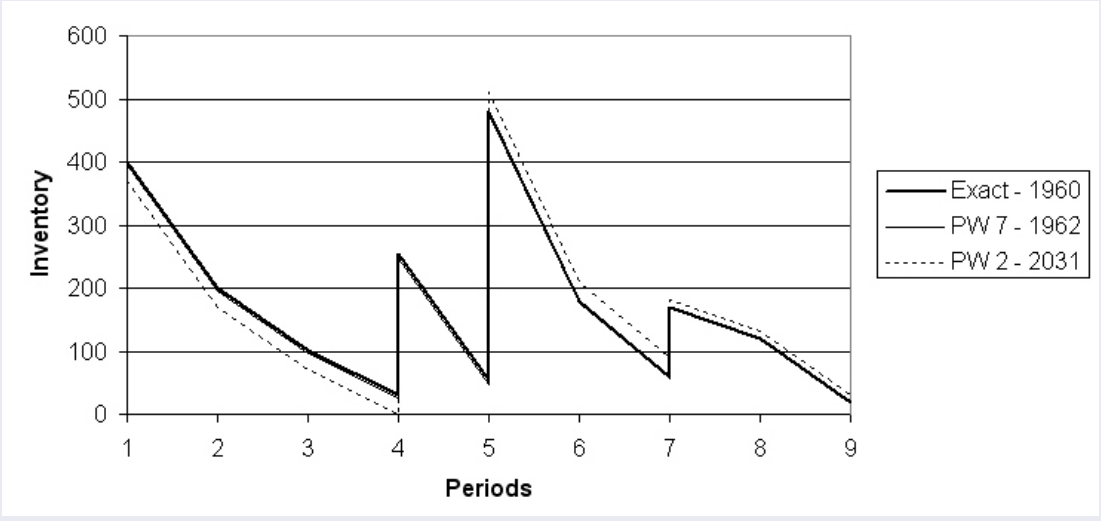

Figure: $h = 1$, $a = 250$, $s = 10$, $v = 0$, $\tau = 0.2$

objConstraint

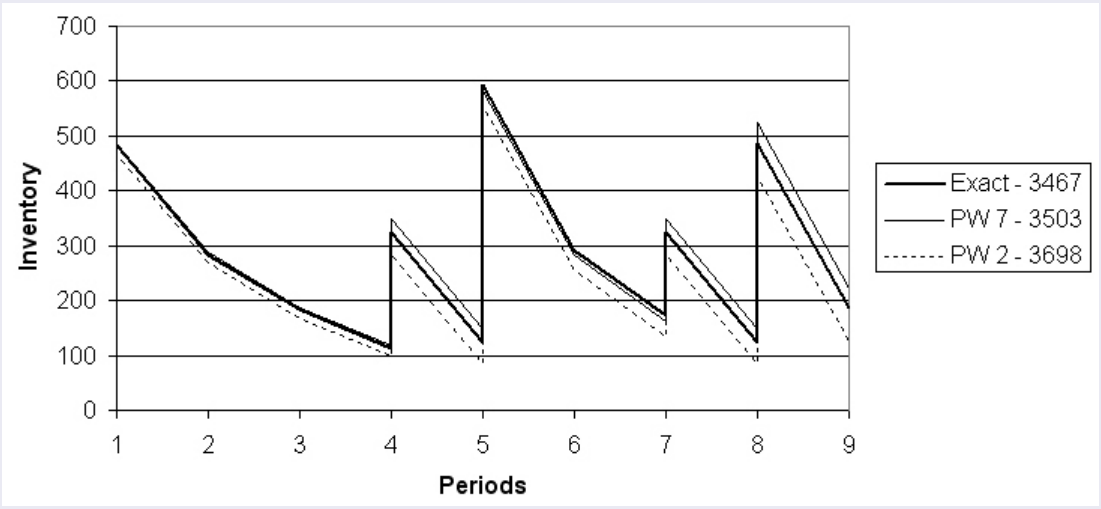

Figure: $h = 1$, $a = 350$, $s = 50$, $v = 0$, $\tau = 0.3$

Generating good LB during the search

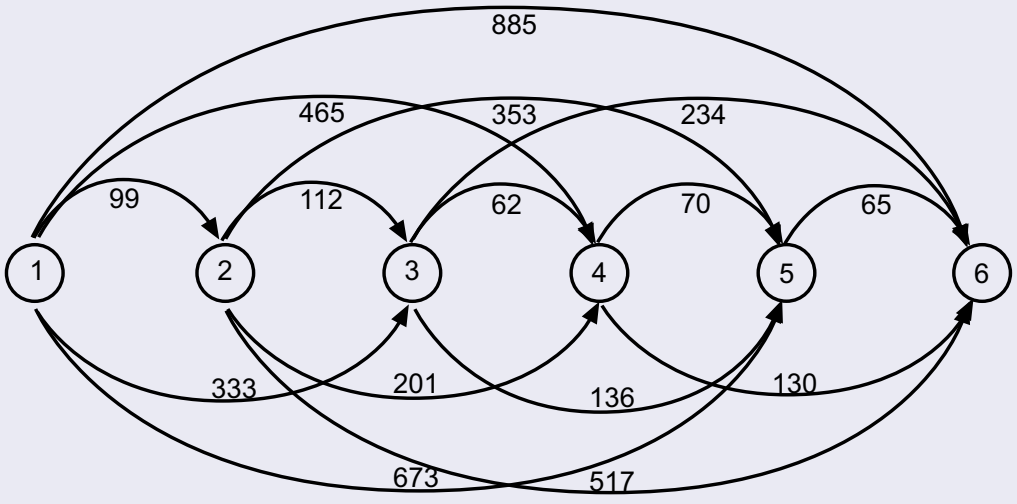

Figure: Shortest Path Relaxation

Generating good LB during the search

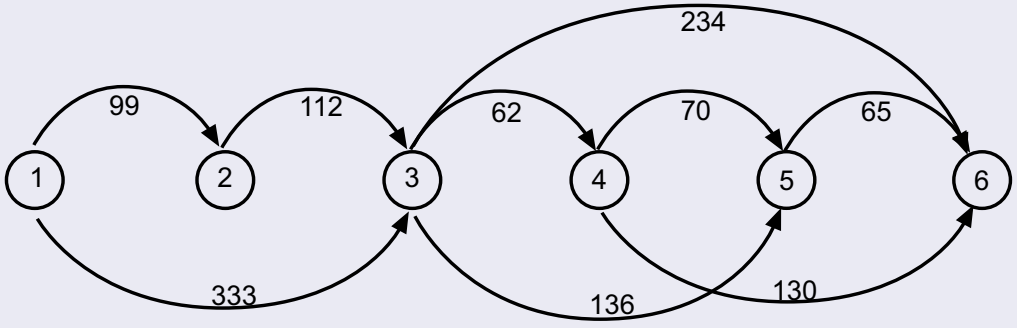

Figure: $\delta_3 = 1$

Generating good LB during the search

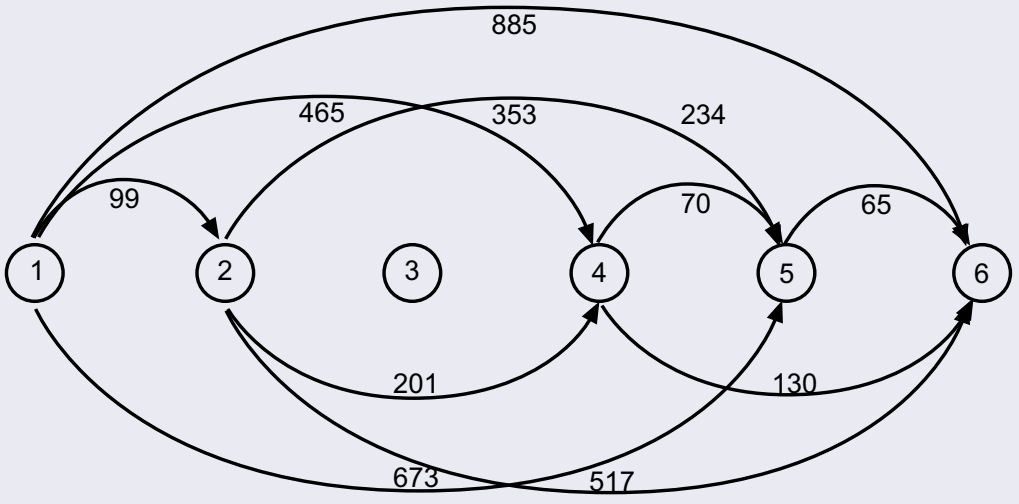

Figure: $\delta_3 = 0$

Experimental results

		Test Set P1				Test Set P2			
		$\sigma_t/\tilde{d}_t = 1/3$		$\sigma_t/\tilde{d}_t = 1/6$		$\sigma_t/\tilde{d}_t = 1/3$		$\sigma_t/\tilde{d}_t = 1/6$	
a	N	$s = 15$	$s = 25$	$s = 15$	$s = 25$	$s = 15$	$s = 25$	$s = 15$	$s = 2$
50	20	0,150	0,030	0,020	0,020	0,030	0,040	0,050	0,050
	22	−−	−−	0,020	0,030	0,040	0,030	0,060	0,060
	24	0,031	0,040	0,030	0,030	0,060	0,040	0,080	0,070
	26	0,040	0,070	0,040	0,040	0,060	0,050	0,120	0,120
	28	0,050	0,080	0,060	0,050	0,070	0,060	0,170	0,121
	30	0,080	0,090	0,060	0,050	0,080	0,081	0,161	0,161
	32	0,100	0,090	0,070	0,081	0,120	0,141	0,180	0,150
	34	−−	−−	0,060	0,070	0,140	0,080	0,180	0,160
	36	0,210	0,111	0,080	0,081	0,161	0,090	0,230	0,180
	38	0,171	0,100	0,090	0,080	0,140	0,120	0,210	0,241
100	20	0,030	5,949	0,020	0,030	0,040	0,030	0,020	0,020
	22	0,030	0,030	0,030	0,030	0,040	0,030	0,031	0,030
	24	0,030	0,040	0,040	0,030	0,040	0,041	0,040	0,030
	26	0,040	0,040	0,040	0,040	0,080	0,050	0,050	0,050
	28	0,060	0,070	0,050	0,050	0,060	0,071	0,060	0,051
	30	0,061	0,060	0,060	0,060	0,071	0,080	0,061	0,080
	32	0,080	−−	0,070	0,070	0,081	0,090	0,071	0,070
	34	0,070	0,060	0,070	0,070	0,090	0,080	0,231	0,070

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This thesis brings contributions to two fields

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  - Global Chance-Constraints
  - Optimization-Oriented Global Chance-Constraints

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- Stochastic Constraint Programming
  - Global Chance-Constraints
  - Optimization-Oriented Global Chance-Constraints
- Stochastic Inventory Control
  - Application of the former techniques
  - Improved formulations for computing (R^n, S^n) policy parameters

In summary

- We proposed *novel optimization models and algorithms that constitute a step forward in stochastic inventory control*

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- We proposed *novel optimization models and algorithms that constitute a step forward in stochastic inventory control*
- We made *theoretical contributions to a new trend of research that applies constraint reasoning* — a technique that in the last 25 years generated a remarkable amount of lore — *to optimization problems under uncertainty*

The End

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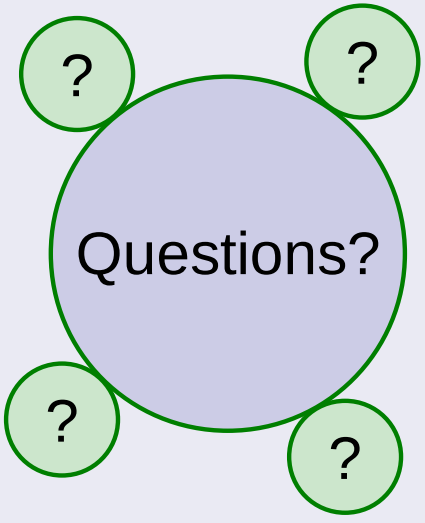

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